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SELECT
Mechanical Exercises:

Shewing how to construct different
CLOCKS, ORRERIES, and SUN-DIALS;
ON PLAIN AND EASY PRINCIPLES.

WITH SEVERAL
MISCELLANEOUS ARTICLES;
A N D
NEW TABLES,

- I. For expeditiously computing the Time of any NEW or FULL MOON within the Limits of 6000 Years before and after the 18th Century.
- II. For graduating and examining the usual Lines on the SECTOR, PLAIN SCALE, and GUNTER.

Illustrated with COPPER-PLATES.

To which is prefixed,

A short Account of the Life of the Author.

By JAMES FERGUSON, F. R. S.

L O N D O N:

Printed for W. STRAHAN: and T. CADELL, in the Strand.

MDCCLXXIII.

DATA

Metabolic Homeostasis

Homeostatic control systems

Regulation of blood glucose levels

Regulation of blood glucose levels

Homeostasis

Regulation of blood glucose levels

Regulation

Regulation of blood glucose levels

Want you to explain the following:
What is meant by homeostasis and how
does it relate to blood glucose levels?
What is the primary regulator of blood glucose
levels and how does it regulate blood glucose
levels? What are the other factors that
influence blood glucose levels?

Regulation of blood glucose levels

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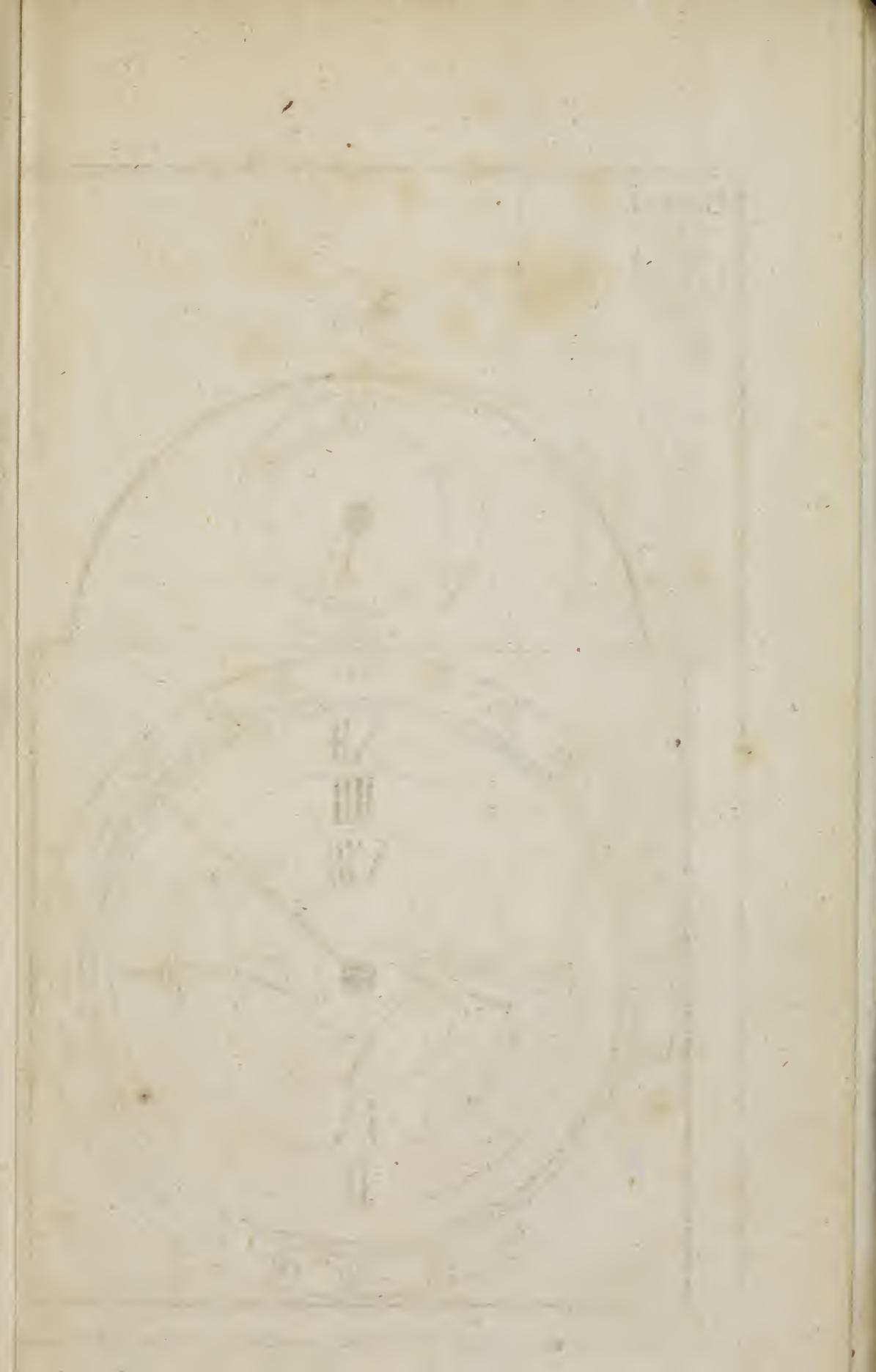
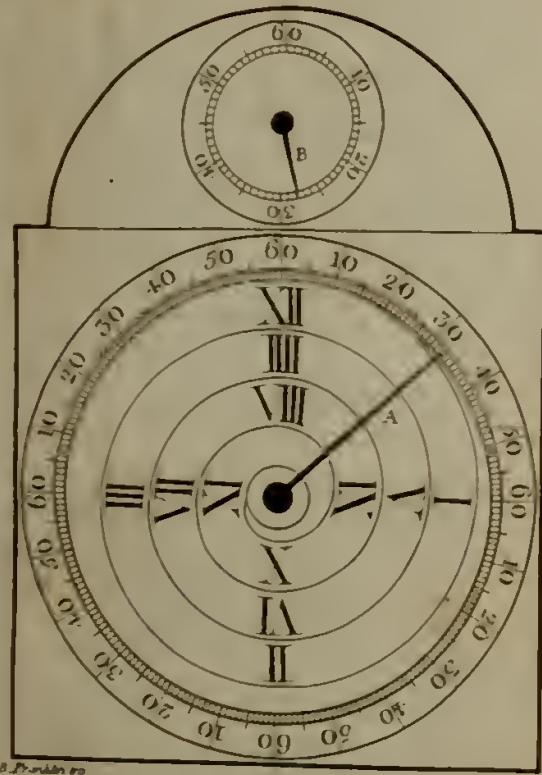


PLATE I.

Fig. 1.



D. B. Bradbury

Fig. 2.

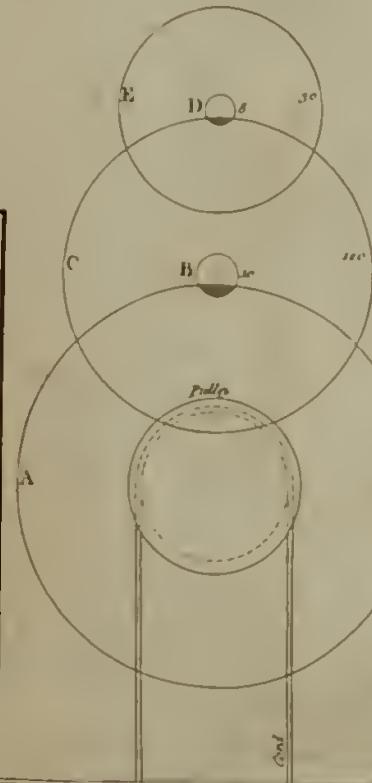
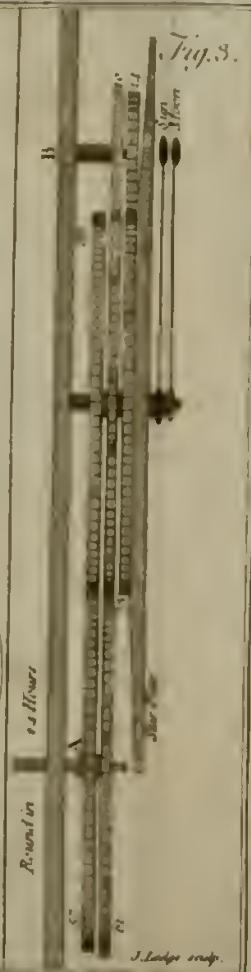
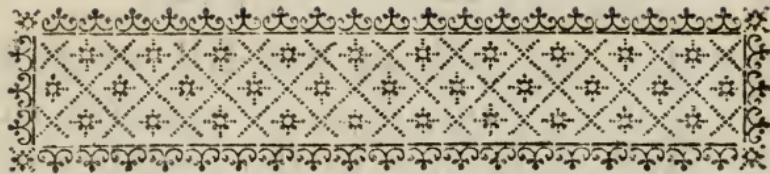


Fig. 3.



J. Lander engr.



i/c

S E L E C T
E X E R C I S E S.

A Clock shewing the Hours, Minutes, and Seconds, having only three Wheels and two Pinions in the whole Movement. Invented by Dr. FRANKLIN of Philadelphia.

THE dial-plate of this clock is represented by Fig. 1. of Plate I. The hours are engraven in spiral spaces, along two diameters of a circle containing four times 60 minutes. The index *A* goes round in four hours, and counts the minutes from any hour it has passed by, to the next following hour. The time, as it appears in the figure, is either $32\frac{1}{2}$ B minutes

minutes past XII, or past III, or past VIII; and so on, in each quarter of the circle, pointing to the number of minutes after the hours the index last left in its motion. Now, as one can hardly be four hours mistaken in estimating the time, he can always tell the true hour and minute, by looking at the clock, from the time he rises till the time he goes to bed. The small hand *B*, in the arch at top, goes round once in a minute, and shews the seconds, as in a common clock.

Fig. 2. shews the wheel-work of this clock. *A* is the first or great wheel, it contains 160 teeth, goes round in four hours, and the index *A* (Fig. 1.) is put upon its axis, and moved round in the same time: The hole in the index is round, it is put tight upon the round end of the axis, so as to be carried by the motion of the wheel, but may be set at any time to the proper hour and minute, without affecting either the wheel or its axis. This wheel of 160 teeth turns a pinion

B of

B of 10 leaves; and as 10 is but a 16th part of 160, the pinion goes round in a quarter of an hour. On the axis of this pinion is the wheel *C* of 120 teeth; it also goes round in a quarter of an hour, and turns a pinion *D*, of 8 leaves, round in a minute; for there are 15 minutes in a quarter of an hour, and 8 times 15 is 120. On the axis of this pinion is the *second-hand* *B* (Fig. 1.) and also the common wheel *E* (Fig. 2.) of 30 teeth, for moving a pendulum (by pallets) that vibrates seconds, as in a common clock.

This clock is not designed to be wound up by a winch, but to be drawn up like a clock that goes only 30 hours. For this purpose, the line must go over a pulley on the axis of the great wheel, as in a common 30 hour clock. Several clocks have been made according to this ingenious plan of the Doctor's; and I can affirm, that I have seen one of them, which measures time exceedingly well.—

The simpler that any machine is, the better it will be allowed to be, by every man of science.

Another Clock that shews the Hours, Minutes, and Seconds, by means of only three Wheels and two Pinions in the whole Movement.

As Dr. FRANKLIN, whom I rejoice to call my friend, is perhaps the last person in the world, who would take any thing amiss that looks like an amendment or improvement of any scheme he proposes, I have ventured to offer my thoughts concerning his clock, and how one might be made as simple as his, with some advantages. But I must confess, that my alteration is attended with some inconveniences, of which his are entirely free.—I shall mention both, to the best of my knowledge, that they who chuse to have such simple and cheap

PLATE II.

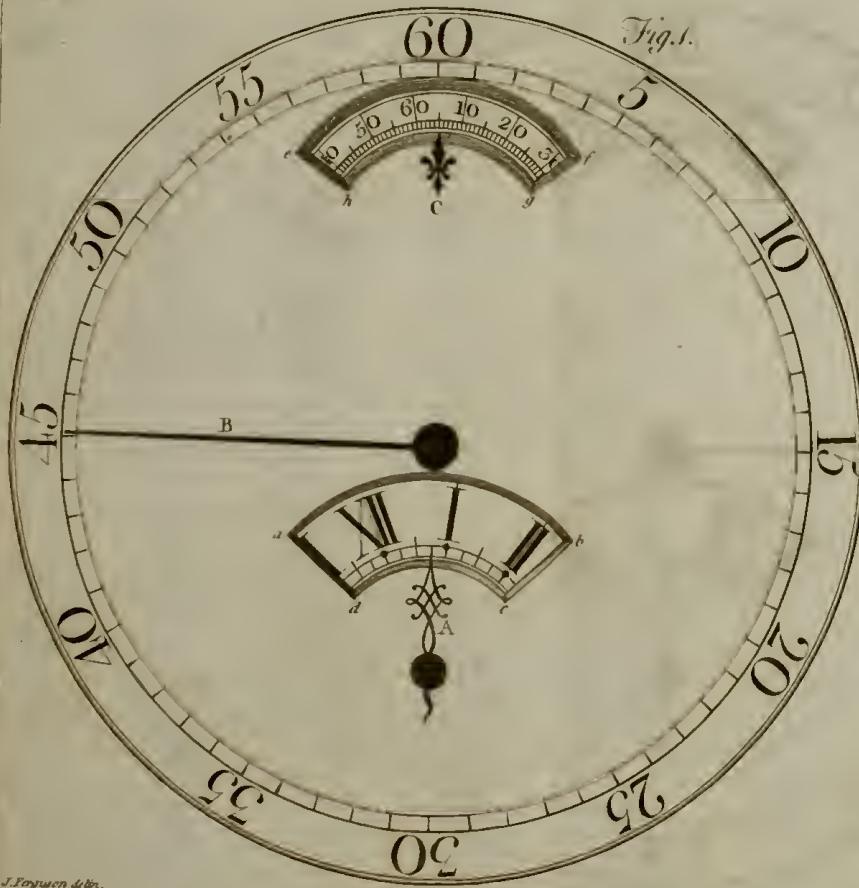


Fig. 1.

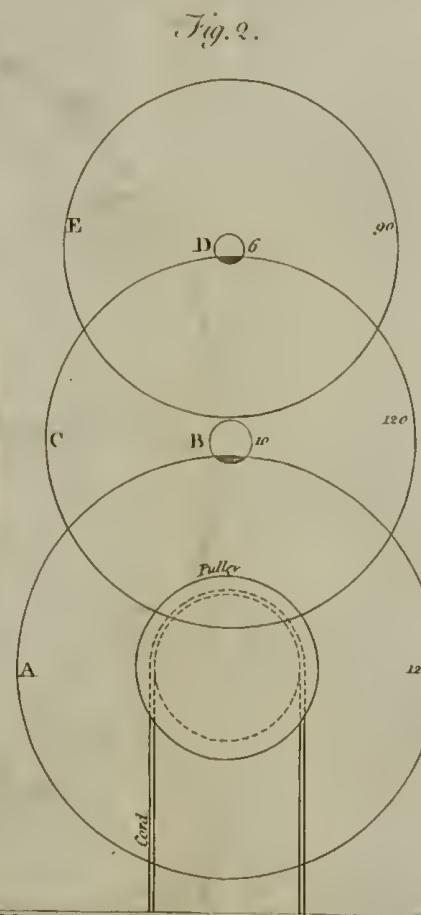


Fig. 2.

J. Forstman *et al.*

J. L. C. sculp.

cheap clocks may have them made in either way they please.

The Doctor's clock cannot well be made to go a week without drawing up the weight; and if a person wakes in the night, and looks at the clock, he may possibly be mistaken four hours in reckoning the time by it, as the hand cannot be upon any hour, or pass by any hour, without being upon or passing by four hours at the same time. To avoid these inconveniences, I have thought of the following method.

In Fig. 1. of Plate II. the dial-plate of such a clock is represented; in which there is an opening *abcd* below the center. Through this opening, part of a flat plate appears, on which the twelve hours are engraved, and divided into quarters. This plate is contiguous to the back of the dial-plate, and turns round in 12 hours; so that the true hour, or part thereof, appears in the middle of the opening,

at the point of an index *A*, which is engraved on the face of the dial-plate. —*B* is the minute-hand, as in a common clock, going round through all the 60 minutes on the dial in an hour; and, in that time, the plate seen thro' the opening *ab* *c* *d* shifts one hour under the fixt engraved Index *A*. By these, you always know the hour and minute, at whatever time you view the dial-plate.—In this Plate is another opening *efgb*, thro' which the seconds are seen on a flat moveable ring, almost contiguous to the back of the dial-plate; and, as the ring turns round, the seconds upon it are shewn by the top-point of a *fleur-de-lis* *C*, engraved on the face of the dial-plate.

Fig. 2. represents the wheels and pinions in this clock. *A* is the first or great wheel; it contains 120 teeth, and turns round in 12 hours. On its axis is the plate on which the 12 hours above mentioned are engraved. This plate

plate is not fixed on the axis, but is only put tight upon a round part thereof, so that any hour, or part of an hour, may be set to the top of the fixed index *A* (Plate I.) without affecting the motion of the wheel. For this purpose, twelve small holes are drilled through the plate, one at each hour, among the quarter divisions: and, by putting a pin into any hole in view, the plate may be set, without affecting any part of the wheel-work. This great wheel *A*, of 120 teeth, turns a pinion *B* of 10 leaves round in an hour; and the minute-hand *B* (Fig. 1.) is on the axis of this pinion, the end of the axis not being square, but round; that the minute-hand may be turned occasionally upon it, without affecting any part of the movement. On the axis of the pinion *B* is a wheel *C* of 120 teeth, turning round in an hour, and turning a pinion *D* of 6 leaves in 3 minutes; for 3 minutes is a 20th part of an hour, and 6

is a 20th part of 120. On the axis of this pinion is a wheel *E* of 90 teeth, going round in 3 minutes, and keeping a pendulum in motion that vibrates seconds, by pallets, as in a common clock, where the pendulum wheel has only 30 teeth, and goes round in a minute.— But, as this wheel goes round only in 3 minutes, if we want it to shew the seconds, a thin plate must be divided into 3 times 60, or 180 equal parts, and numbered 10, 20, 30, 40, 50, 60; 10, 20, 30, 40, 50, 60; 10, 20, 30, 40, 50, 60; and fixed upon the same axis with the wheel of 90 teeth, so near the back of the dial-plate, as only to turn round without touching it: and these divisions will shew the seconds, thro' the opening *efgh* in the dial-plate, as they slide gradually round below the point of the fixed *fleur-de-lis* *C*.

As the great wheel *A*, and pulley on its axis over which the cord goes (as in a common 30 hour clock) turns round

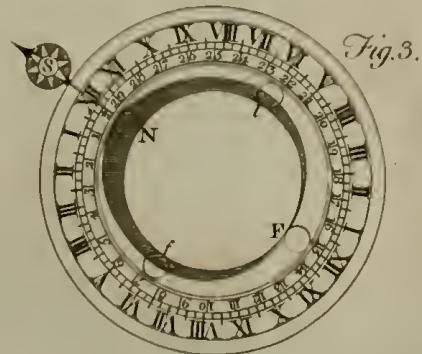
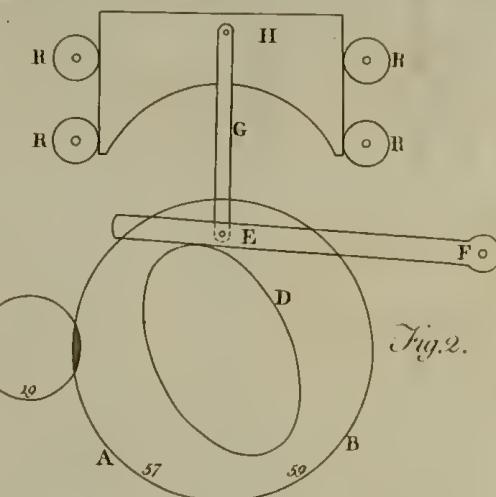
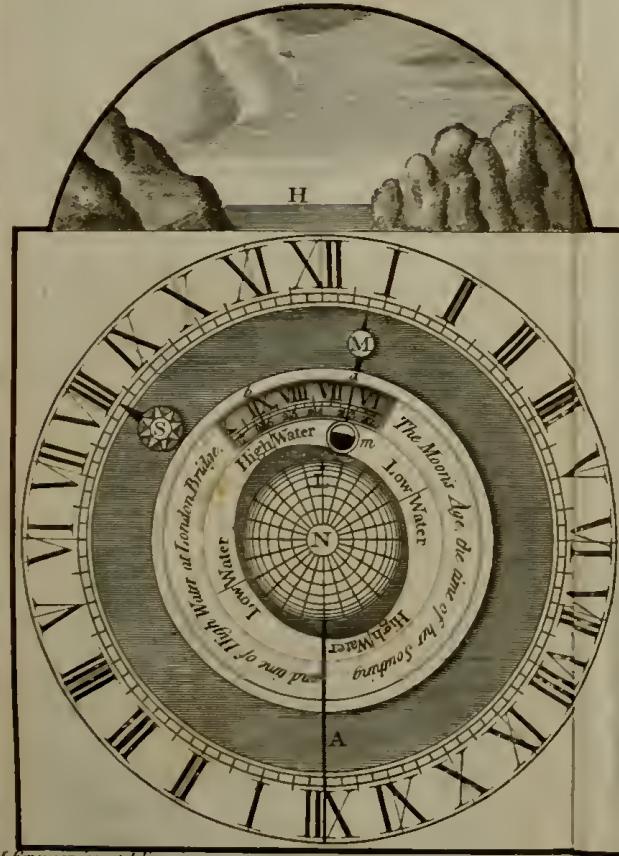
round only once in 12 hours, this clock will go a week with a cord of common length, and always have the true hour, or part of that hour, in sight at the upper end of the fixed index *A* on the dial-plate. These are two advantages it has beyond Dr. Franklin's clock: but it has two disadvantages of which his clock is free. For, in this, although the 12 hour wheel turns the minute-index *B*, yet, if *that* index be turned by hand, to set it to the proper minute for any time, it will not move the 12 hour plate to set the corresponding part of the hour even with the top of the index *A*: and therefore, after having set the minute-index *B* right by hand, the hour-plate must be set right by means of a pin put into the small hole in the plate just below the hour. 'Tis true, there is no great matter in this; but I have some suspicion that the pendulum wheel *E* having 90 teeth instead of the common number 30, may be some disadvantage

disadvantage to the escapement, on account of the smallness of the teeth; and 'tis certain, that it will cause the pendulum-ball to describe but small arcs in its vibrations. Indeed some men of science think small arcs are best; but if they really are, I must confess myself ignorant of the reason. For, whether the ball describes a large or a small arc, if the arc be nearly *cycloidal*, the vibrations will be performed in equal times; the time then depending entirely on the length of the pendulum-rod, not on the length of the arc the ball describes. The larger the arc is, the greater is the *momentum* of the ball; and the greater the *momentum* is, the less will the times of the vibrations be affected by any unequal impulse of the pendulum-wheel upon the pallets.

But the worst thing about this clock (and what every one will allow to be a disadvantage) is, that the weight of the flat ring on which the seconds are engraved,

PLATE III.

Fig. 1.



engraved, will load the pivots of the axis of the pendulum-wheel with a great deal of friction, which ought by all possible means to be avoided ; and yet I have seen one of these clocks (lately made) that goes very well, notwithstanding the weight of this ring. For my own part, I think it might be quite left out ; for seconds are of very little use in common clocks not made for astronomical observations ; and table-clocks never have them.

A Clock shewing the apparent daily Motions of the Sun and Moon, the Age and Phases of the Moon, with the Time of her coming to the Meridian, and the Times of High and Low Water, by having only two Wheels and a Pinion added to the common Movement.

The dial-plate of this clock is represented by Fig. 1. of Plate III. It contains

contains all the 24 hours of the day and night. *S* is the Sun, which serves as an hour-index, by going round the dial-plate in 24 hours ; and *M* is the Moon, which goes round in 24 hours $50\frac{1}{2}$ minutes, from any point in the hour-circle to the same point again, which is equal to the time of the Moon's going round in the heavens, from the meridian of any place to the same meridian again. The Sun is fixed to a circular plate (as Fig. 3.) and carried round by the motion of that plate, on which the 24 hours are engraved, and within them is a circle divided into $29\frac{1}{2}$ equal parts for the days of the Moon's age, accounted from the time of any new Moon to the next after ; and each day stands directly under the time (in the 24 hour circle) of the Moon's coming to the meridian, the XII under the Sun standing for mid-day, and the opposite XII for midnight.—Thus, when the Moon is 8 days old, she comes to the meridian

dian at half an hour past VI in the afternoon; and when she is 16 days old, she comes to the meridian at 1 o'clock in the morning. The Moon *M.* (Fig. 1.) is fixed to another circular plate, of the same diameter with that which carries the Sun; and this Moon-plate turns round in 24 hours $50\frac{1}{2}$ minutes. It is cut open; so as to shew some of the hours, and days of the Moon's age, on the plate below it that carries the Sun, and, across this opening, at *a* and *b* are two short pieces of small wire in the Moon-plate. The wire *a* shews the day of the Moon's age, and time of her coming to the meridian, on the plate below it that carries the Sun; and the wire *b* shews the time of high water for that day, on the same plate. These wires must be placed as far from one another, as the time of the Moon's coming to the meridian differs from the time of high water at the place where the clock is intended to serve.—At London-bridge, it

it is high water when the Moon is two hours and an half past the meridian.

Above this plate, that carries the Moon, there is a fixed plate *N*, supported by a wire *A*, the upper end of which is fixed to that plate, and the lower end is bent to a right angle, and fixed into the dial-plate at the lower-most or midnight XII. This plate may represent the Earth, and the dot at *L* London, or any other place at which the clock is designed to shew the times of high and low water.

Around this plate is an elliptical shade upon the plate that carries the Moon *M*: the highest points of this shade are marked *High Water*, and the lowest points *Low Water*. As this plate turns round below the fixed plate *N*, the high and low water points come successively even with *L*, and stand just over it at the times when it is high or low water at the given place; which times are pointed out by the Sun *S*, among the 24 hours on the dial-plate: and,

and, in the arch of this plate, above XII at noon, is a plate *H* that rises and falls as the tide does at the given place. Thus, when it is high water (suppose at London) one of the highest points of the elliptical shade stands just over *L*, and the tide-plate *H* is at its greatest height: and when it is low water at London, one of the lowest points of the elliptical shade stands over *L*, and the tide-plate *H* is quite down, so as to disappear beyond the dial-plate.

As the Sun *S* goes round the dial-plate in 24 hours, and the Moon *M* goes round it in 24 hours $50\frac{1}{2}$ minutes, the Moon goes so much slower than the Sun as to make only $28\frac{1}{2}$ revolutions in the time the Sun makes $29\frac{1}{2}$; and therefore the Moon's distance from the Sun is continually changing; so that, at whatever time the Sun and Moon are together, or in conjunction, in $29\frac{1}{2}$ days afterward they will be in conjunction again. Consequently, the plate that carries the Moon moves so

much flower than the plate that carries the Sun, as always to make the wire a shift over one day of the Moon's age on the Sun's plate in 24 hours.

In the plate that carries the Moon, there is a round hole *m*, thro' which the phase or appearance of the Moon is seen on the Sun's plate, for every day of the Moon's age from change to change. When the Sun and Moon are in conjunction, the whole space seen through the hole *m* is black: when the Moon is opposite to the Sun (or full) all that space is white: when she is in either of her quarters, the same space is half black half white: and different in all other positions, so as the white part may resemble the visible or enlightened part of the Moon for every day of her age.

To shew these various appearances of the Moon, there is a black shaded space (Fig. 3.) as *NFFl*, on the plate that carries the Sun. When the Sun and Moon are in conjunction, the whole

whole space seen through the round hole is black, as at *N*: when the Moon is full, opposite to the Sun, all the space seen through the round hole is white, as at *F*: when the Moon is in her first quarter, as at *f*, or in her last quarter, as at *l*, the hole is only half shaded; and more or less accordingly for each position of the Moon with regard to her age; as is abundantly plain by the Figure.

The wheel-work and tide-work of this clock is represented by Fig. 2. in which *A* and *B* are two wheels of equal diameters. *A* has 57 teeth, its axis is hollow, it comes through the dial of the clock, and carries the Sun-plate with the Sun (*S*, in Fig. 1.). *B* has 59 teeth, its axis is a solid spindle, turning within the hollow axis of *A*, and carrying the Moon-plate with the Moon (*M*, in Fig. 1.). A pinion *C* of 19 leaves takes into the teeth of both the wheels, and turns them round. This pinion is turned round, by the common clock-work, in 8 hours; and,

as 8 is a third part of 24, so 19 is a third part of 57: and therefore the wheel *A*, of 57 teeth, that carries the Sun, will go round in 24 hours exactly. But, as the same pinion *C* (that turns the wheel *A* of 57 teeth) turns also the wheel *B* of 59 teeth, this last wheel will not turn round in less than 24 hours $50\frac{1}{2}$ minutes of time: for as 57 teeth are to 24 hours, so are 59 teeth to 24 hours $50\frac{1}{2}$ minutes, very nearly.

On the back of the Moon-wheel of 59 teeth is fixed an elliptical ring *D*, which, as it turns round, raises and lets down a lever *E F*, whose center of motion is on a pin at *F*; and this, by means of an upright bar *G*, raises and lets down the tide-plate *H*, twice in the time of the Moon's revolving from the meridian to the meridian again. The upper edge of this plate is shewn at *H*, in Fig. 1. and it moves between four rollers *R*, *R*, *R*, *R*, in Fig. 2.

I have made one of these clocks to go by the movement of an old watch, in the following manner:

The



PLATE IV.

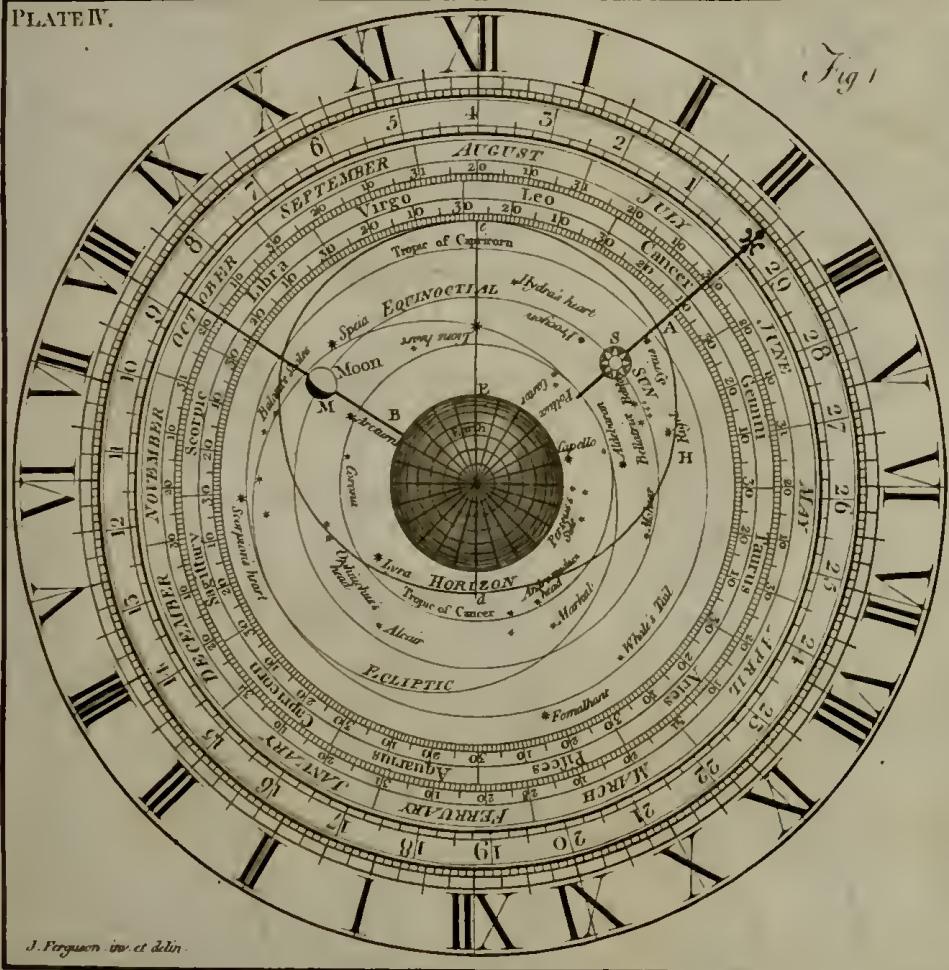
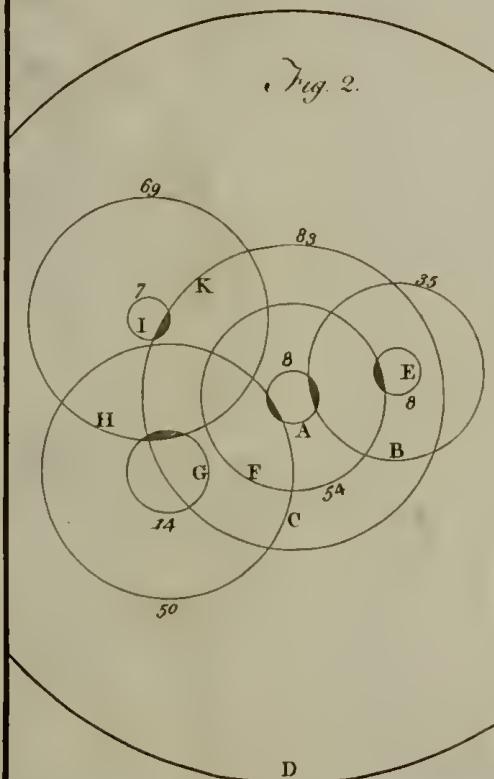


Fig. 2.



J. Ferguson inv. et delin.

The first, or great wheel of a watch, goes round in four hours. I put a wheel of 20 teeth on the end of the axis of that wheel, to turn a wheel of 40 teeth on the axis of the pinion *C*; by which means, *that* pinion is turned round in 8 hours, the wheel *A* in 24 hours, and the wheel *B* in 24 hours $50\frac{1}{2}$ minutes.—I never saw nor heard of any other clock of this kind.

An astronomical Clock, shewing the apparent daily Motions of the Sun, Moon, and Stars, with the Times of their rising, southing, and setting, the Places of the Sun and Moon in the Ecliptic, and the Age and Phases of the Moon, for every Day of the Year.

The dial-plate of this clock is represented by Fig. 1. of Plate IV. It contains all the 24 hours of the day and

night, and each hour is divided into 12 equal parts, so that each part answers to 5 minutes of time.

Within these divisions of the hour-circle is a flat ring, the face of which is just even (or in the same plane) with the face of the hour-circle. This ring is divided into $29\frac{1}{2}$ equal parts (numbered 1, 2, 3, 4, &c. from the right hand toward the left) which are the days of the Moon's age from change to change: the ring turns round in 24 hours, and has a *fleur-de-lis* upon it, serving as an hour-index to point out the time of the day or night in the 24 hour circle.

Within this ring, and about four tenths of an inch below its flat surface, is a flat circular plate, on which the months and days of the year are engraved; and within these, on the same plate, is a circle containing the signs and degrees of the ecliptic, divided in such a manner, as that each particular

particular day of the year stands over the sign and degree of the Sun's place on that day.

Within this circle, on the same plate, the ecliptic, equinoctial, and tropics, are laid down; and all the stars of the first, second, and third magnitude that are ever seen in the latitude of London, according to their respective right ascensions and declinations; those of the first magnitude being distinguished by eight points, those of the second by six, and those of the third by five. This plate turns round in 23 hours 56 minutes 4 seconds 6 thirds of time, which is the length of a sydereal day; and consequently it makes 366 revolutions (as the stars do in the heavens) in the time the Sun makes 365; the number of sydereal days in a year exceeding the number of solar days by one.

Over the middle of this plate, and about four tenths of an inch from it, is a fixed plate *E*, to represent the

Earth ; round which, the Sun, Moon, and Stars move in their proper times, *viz.* the Sun in 24 hours, the Moon in 24 hours $50\frac{1}{2}$ minutes, and the Stars in 23 hours 56 minutes 4 seconds $\frac{2}{3}$ thirds. The Sun *S* is carried round by a wire *A*, which is fixed into the inside of the Moon's age ring, even with the *fleur-de-lis*; the Moon *M* is carried round by a wire *B*, which is fixed to the axis of a wheel below the Earth *E*, and the Star-plate is turned round by a wheel at the back of the dial-plate.

Over the dial-plate is a glass, as in common clocks. On this glass is an ellipsis *H*, drawn with a diamond, to represent the horizon of the place for which the clock is to serve, and across this horizon is a straight line *e E d* (even with the two XII's) to represent the meridian. All the Stars that are seen at any time within this ellipsis are above the horizon at that time, and all those that are without it are then

then below the horizon. When any Star on the plate comes to the left-hand side of the horizon (the Stars moving from left to right) the like Star in the heavens is rising; when it comes under the meridian line *e E*, the like Star in the heavens is on the meridian of the place; and when it comes to the right-hand side of the horizon, the like Star in the heavens is setting.

When the point of the ecliptic, that the Sun's wire *A* intersects, comes to the left-hand side of the horizon, on any day of the year cut by that wire, the *fleur-de-lis* will be at the time of the Sun's rising, in the 24 hour-circle; and at the time of his setting when the intersection of the Sun's wire and ecliptic comes to the western side of the horizon.—The like is to be understood with regard to the rising and setting of the Moon, when the point of the ecliptic which her wire *B* intersects

fects comes to the left and right hand sides of the horizon.

Every 24 hours, the Moon's wire shifts over one day of her age in the circle of $29\frac{1}{2}$ equal parts on the flat ring above mentioned. Each of these day-spaces is divided into four equal parts, for shewing the Moon's age to every sixth hour thereof. Thus, as the Moon's wire *B* stands in the figure, it shews the Moon to be 8 days 18 hours old. It shifts quite round the ring, and carries the Moon round from the Sun to the Sun again, in 29 days 12 hours and 45 minutes.

The Sun, on its wire *A*, goes 365 times round in 365 days; and, in that time, the Star-plate, with the months and days of the year upon it, goes 366 times round. So that, for every revolution of the Sun, the Star-plate advances forward, under the Sun's wire, through the space of one day in the circle of months: and by this means
the

the Sun's wire shews the day of the month throughout the whole year; and at the same time, for each particular day of the year, it shews the Sun's place in the ecliptic, in the circle of signs.—The Moon's wire *B* cuts the Moon's place in the circle of signs, for every day of her age, throughout the year.

The whole circle of signs shifts round, under the Sun's wire *A*, in 365 days 5 hours 48 minutes 58 seconds, which is the time the Sun takes in going quite round the ecliptic: and the Moon's wire shifts over all the circle of signs in 27 days 7 hours and 43 minutes, which is the time of the Moon's going round the ecliptic. And thus, by these different motions of the Sun and Moon, there are always 29 days 12 hours, and 45 minutes between any conjunction of the Sun and Moon, and the next succeeding one.

The moon *M* is a round ball, half black half white: it turns round its axis,

axis, or wire *B*, in 29 days 12 hours 45 minutes, and so shews all the different phases of the Moon, for every day of her age, from change to change. When the Sun and Moon are together, or in conjunction, the white side of the Moon-ball *M* is toward the Sun, and the black side toward the eye of a spectator looking at the clock, who then can see no part of the white (or apparently enlightened) side of the Moon: when the Moon is full, or opposite to the Sun, all the white side of the ball *M* is turned toward the spectator's eye: and when the Moon is in either of her quarters, or 90 degrees from the Sun, the spectator sees half the black and half the white side of the ball; so that the white part of it then appears like the Moon in her first or last quarter.

Fig. 2. represents (what is called) the dial-work of this clock, or the wheels at the back of the dial-plate, between it and the wheels of the common movement, which are contained
between

between two fixed plates, wherein the pivots of their axes turn in holes.

A long pin or spindle is fixed into the movement-plate next the back of the dial-plate, perpendicular to both these plates. This spindle goes thro' the center of the dial-plate, and has the Earth-plate *E* (Fig. 1.) fixed on the end of it.

On this spindle, and close to the movement-plate in which it is fixed, is a fixed pinion *A* of 8 leaves (Fig. 2.) which take into the teeth of a wheel *B*, and of a wheel *C*: the number of teeth in *B* is 35, and the number in *C* is 50.

These two wheels hang and turn upon a large plate *D*; which, by the common clock movement, is turned round upon the axis of the fixed pinion *A* in 24 hours: and consequently, as these wheels are carried round the fixed pinion, and its leaves take into their teeth, each of them will be so far turned round its axis, in 24 hours,

as

as is equal to 8 of their teeth. The axis of the plate *D* is hollow, and turns upon the solid fixed spindle or axis of the pinion *A*; and the side of the plate *D* that is nearest to the clock-movement is almost contiguous to the end of the pinion *A* next to the dial-plate of the clock. Hence it is plain, that the two wheels *B* and *C*, which are carried round the pinion *A*, and are turned by it, must be on that side of the plate *D* which is almost contiguous to the pinion. All the rest of the wheels and pinions in the figure are on the other side of the plate *D*; namely, on the side of it which is next to the back of the dial-plate.

The axes of the wheels *B* and *C* go through the plate *D*: on the top of the axis *B* is a pinion *E* of 8 leaves, which turns a wheel *F* of 54 teeth. The axis of this wheel is hollow, and turns upon the hollow axis of the plate *D*; it comes through the center of the dial-plate of the clock, and carries

ries the Moon round by the wire *B* in Fig. 1. which wire turns round in a piece that goes tight upon the hollow end of the axis of *F* (Fig. 2.) just under the Earth-plate *E* (Fig. 1.).

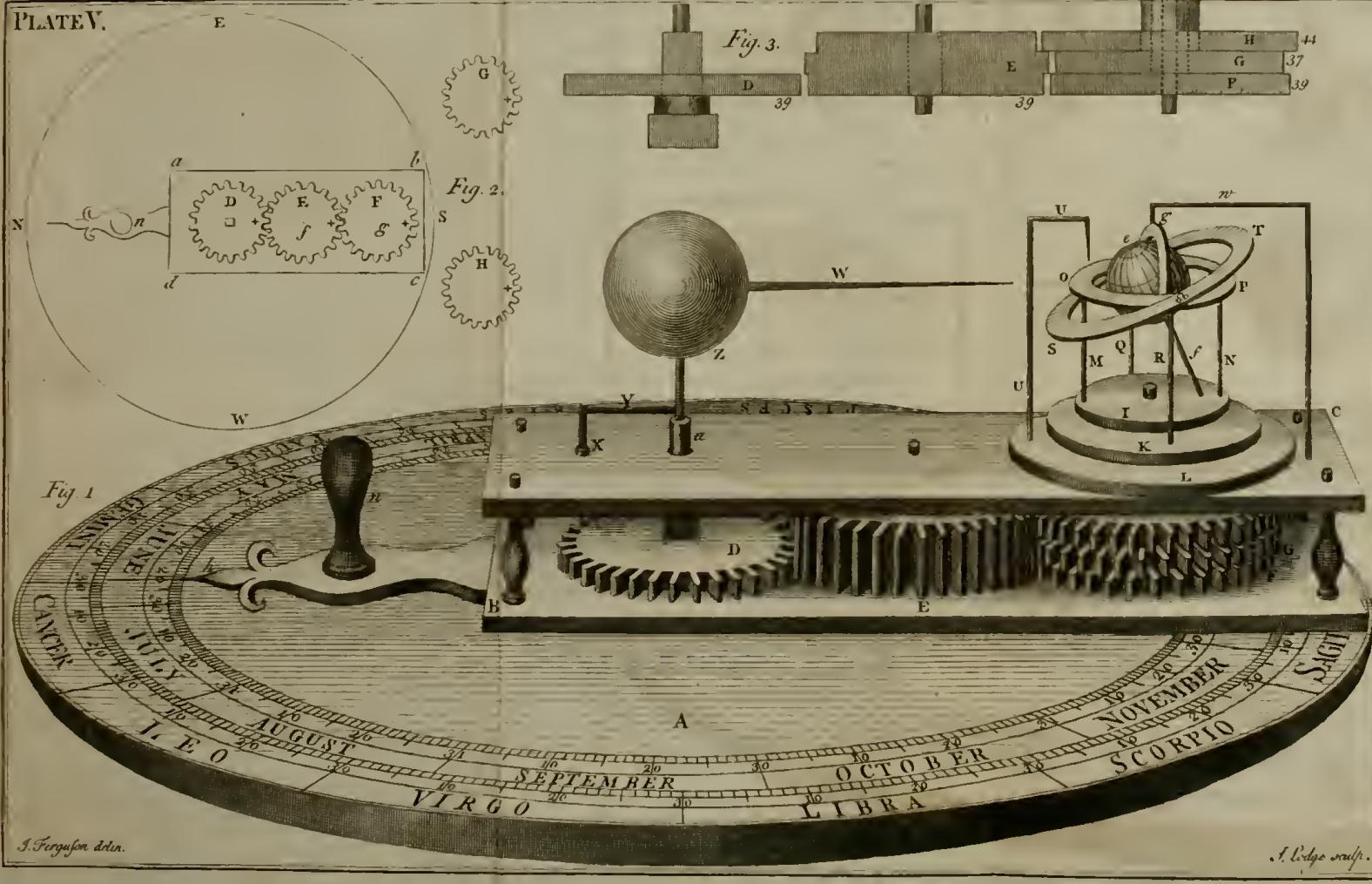
The hollow axis of the plate *D* turns round, as the plate does, in 24 hours. On the end of this axis, just under the middle of the Earth-plate *E*, is a small wheel of 20 teeth, turning a contrate wheel of the same number, the pivots of whose axis turn in the piece that carries the Moon's wire; and this wheel turns another wheel of the same number of teeth, fixed on the Moon's wire or axis. By these wheels (which lie concealed under the Earth-plate *E*) the Moon is turned round her axis in 29 days 12 hours 45 minutes, as above mentioned, and shews her different phases. These three last mentioned wheels are not represented in Fig. 2. because they could not be put in without confusing it.

On

On the axis of the wheel *C* of 50 teeth is a pinion *G* of 14 leaves, which turns a wheel *H* of 69 teeth, on whose axis is a pinion *I* of 7 leaves, turning a wheel *K* of 83 teeth: this wheel is pinned fast to the back of the Star-plate, which, together with the wheel, turns round in a sydereal day, or in 23 hours 56 minutes 4 seconds 6 thirds of mean solar time; and the above-mentioned wheels, which belong to the Moon, will carry her round from the meridian to the meridian again in 24 hours $50\frac{1}{2}$ minutes; if the plate *D*, on which all the wheels hang, be turned round in 24 mean solar hours.

The plate *D* carries the ring of $29\frac{1}{2}$ equal parts round, within the fixed 24 hour-circle of the dial-plate, in 24 hours, by means of four pillars, whose opposite ends are fixed into the plate *D*, and into the ring. By this, the Sun is carried round in 24 hours (its wire *A* being fixed into the ring) and also

PLATE V.



also the *fleur-de-lis* that shews the time, like an index, in the 24 hour-circle.

I contrived this clock above twenty years ago, and made a model of it in wood, which I have still in my custody; and since that time,

Another Clock, for shewing the apparent daily Motions of the Sun, Moon, and Stars; with the Times of their rising, southing, and setting.

The dial-plate of this clock is like the one last described; but the wheel-work at the back of the dial-plate is different, and rather more simple.

Two wheels, one of 57 teeth and the other of 61, are fixed upon an axis *A* (see Fig. 3. of Plate I.) and turned round by the common clock-motion in 24 hours.

The wheel of 57 teeth turns a wheel of 59, on the top of whose axis, above (or

(or before the face of) the Star-plate, is a wire that carries the Moon round, from the meridian to the meridian again, in 24 hours $50\frac{1}{2}$ minutes.

The wheel of 61 teeth turns another of the same number, whose axis is hollow, and turns on the solid axis of the wheel of 59. On the top of this hollow axis, between the Moon's wire and the Star-plate, is a wire that carries the Sun round from the meridian to the meridian again, in 24 mean solar hours.

The last mentioned wheel of 61 teeth (whose axis carries the Sun) turns a wheel of 20 teeth, upon a pin fixed in the immoveable plate *B*; and to this wheel of 20 teeth is fixed a wheel of 24, by pins put through them, and riveted on their outsides: this wheel of 24 teeth turns a wheel of 73, loose on the hollow axis of the Sun-wheel of 61, and riveted fast to the Star-plate; which,

which, together with the said wheel, turns 366 times round in 365 days, measured by the Sun's motion.

How to regulate a Clock by the Motion of the Stars, so as to measure mean solar Time exactly.

As the Stars make 366 revolutions in 365 days, each Star comes to the meridian 3 minutes $55\frac{9}{10}$ seconds on each succeeding day or night sooner than it did on the day or night before. We shall first give the following table of their revolutions for a year, and then shew its use in regulating clocks and watches, so as to keep mean solar time.

A TABLE for regulating Clocks and Watches to true equal Time, by the daily Revolutions of the Stars.

Rev.	Days.	H.	M.	S.	H.	M.	S.
1	20	23	56	4.1	0	3	55.9
2	1	23	52	8.2	0	7	51.8
3	2	23	48	12.3	0	11	47.7
4	3	23	44	16.4	0	15	43.6
5	4	23	40	20.5	0	19	39.5
6	5	23	36	24.6	0	23	35.4
7	6	23	32	28.7	0	27	31.3
8	7	23	28	32.8	0	31	27.2
9	8	23	24	36.9	0	35	23.1
10	9	23	20	41.0	0	39	19.0
11	10	23	16	45.1	0	43	14.9
12	11	23	12	49.2	0	47	10.8
13	12	23	8	53.3	0	51	6.7
14	13	23	4	57.4	0	55	2.6
15	14	23	1	1.5	0	58	58.5
16	15	22	57	5.6	1	2	54.4
17	16	22	53	9.7	1	6	50.3
18	17	22	49	13.8	1	10	46.2
19	18	22	45	17.9	1	14	42.1
20	19	22	41	22.0	1	18	38.0
21	20	22	37	26.1	1	22	33.9
22	21	22	33	30.2	1	26	29.8
23	22	22	29	34.3	1	30	25.7
24	23	22	25	38.4	1	34	21.6
25	24	22	21	42.5	1	38	17.5
26	25	22	17	46.6	1	42	13.4
27	26	22	13	50.7	1	46	9.3
28	27	22	9	54.8	1	50	5.2
29	28	22	5	58.9	1	54	1.1
30	29	22	2	3.0	1	57	57.0
40	39	21	22	44.0	2	37	16.0
50	49	20	43	25.0	3	16	35.0
60	59	20	4	6.0	3	55	54.0
70	69	19	24	47.0	4	35	13.0
80	79	18	45	28.0	5	14	32.0
90	89	18	6	9.0	5	53	51.0
100	99	17	26	50.0	6	33	10.0
200	199	10	53	40.0	13	6	20.0
300	299	4	20	30.0	19	39	30.0
360	359	0	24	36.0	23	35	24.0
365	364	0	4	56.5	23	55	3.5
366	365	0	1	0.6	23	58	59.4

The

The first column denotes the number of revolutions of the Stars, from the meridian to the meridian again, in a common year of 365 days: the next other columns (titled *Days. H. M. S.*) shew the times in which these revolutions are made: and those in the right-hand part of the Table (titled *H. M. S.*) shew how much any Star gains daily upon the time shewn by a well-regulated clock or watch.

Therefore, to know whether the clock goes true or not, observe the time by the clock when any Star disappears behind a chimney (or any other fixed object) as seen through a hole in a thin plate of metal fixed in a window-shutter; and if the same Star disappears on every succeeding night as much sooner by the clock as to agree with the times shewn in the right-hand part of the Table (as suppose 39 minutes 19 seconds in 10 days, or 1 hour 18 minutes 38 seconds in 20

days) the clock goes true: otherwise it does not, and must be regulated accordingly, by screwing up or letting down the ball of the pendulum, as it goes too slow or too fast.

To find the Length of a Pendulum that shall make any given Number of Vibrations in a Minute, and vice versa.

A pendulum whose length is 39.2 inches, from the point of suspension to the center of oscillation, makes 60 vibrations in a minute; and this is called the standard length. Then, for any other number of vibrations in a minute, say, As the square of the given number of vibrations is to the square of 60, so is the length of the standard to the length sought.—Thus, suppose the given number of vibrations to be 30 *per minute*: the square of 30 is 900, and the square of 60 is 3600: then, as

as 900 is to 3600, so is 39.2 to 156.8; so that the length required for 30 vibrations *per minute* is 156.8 inches.

If the length of the pendulum be given, and the number of vibrations it makes in a minute be required; say, As the given length is to the standard length (39.2 inches) so is the square of 60 vibrations to the square of the number required: the square root of which shall be the number of vibrations made by the pendulum in a minute. Thus, suppose the given length to be 156.8 inches: As 156.8 is to 39.2, so is 3600 (the square of 60) to 900; the square root of which is 30, the number of vibrations that this pendulum will make in a minute.

The length of a pendulum that would make only one vibration in a minute is 3920 yards, or 141120 inches: and the length of a pendulum that would make 240 vibrations in a minute (or 4 in a second) is 2.45 inches.

In these calculations it is supposed that the weight of the pendulum-rod bears little or no sensible proportion to the weight of the ball. But as this cannot be the case in practice, the center of oscillation will always be further from the point of suspension than the calculation makes it; and this must be found by trial.

To divide the Circumference of a Circle into any given Number of equal Parts, whether even or odd.

As there are very uncommon and odd numbers of teeth in some of the wheels of astronomical clocks, and which consequently could not be cut by any common engine used by clock-makers for cutting the numbers of teeth in their clock-wheels, I thought proper to shew how to divide the circumference of a circle into any given odd or even number of equal parts,

so as *that* number may be laid down upon the dividing plate of a cutting engine.

There is no odd number, but from which, if a certain number be subtracted, there will remain an even number, easy to be subdivided. Thus, supposing the given number of equal divisions of a circle on the dividing plate to be 69; subtract 9, and there will remain 60.

Every circle is supposed to contain 360 degrees: therefore say, As the given number of parts in the circle, which is 69, is to 360 degrees, so is 9 parts to the corresponding arc of the circle that will contain them: which arc, by the Rule of Three, will be found to be $46\frac{9}{100}$. Therefore, by the line of chords on a common scale, or rather on a sector, set off $46\frac{9}{100}$ (or $46\frac{9}{10}$) degrees with your compasses, in the periphery of the circle, and divide that arc or portion of the circle into 9 equal parts, and the rest of the circle

into 60; and the whole will be divided into 69 equal parts, as was required.

Again, suppose it were required to divide the circumference of a circle into 83 equal parts; subtract 3, and 80 will remain.—Then, as 83 parts are to 360 degrees, so (by the Rule of Proportion) are 3 parts to 13 degrees and one hundredth part of a degree; which small fraction may be neglected. Therefore, by the line of chords, and compasses, set off 13 degrees in the periphery of the circle, and divide that portion or arc into 3 equal parts, and the rest of the circle into 80; and the thing will be done.

Once more, suppose it were required to divide a given circle into 365 equal parts: subtract 5, and 360 will remain. Then, as 365 parts are to 360 degrees, so are 5 parts to $4\frac{9}{16}$ degrees. Therefore, set off $4\frac{9}{16}$ (or $4\frac{9}{8}$) degrees in the circle; divide that space into 5 equal parts, and the rest of the circle into 360; and the whole will be divided

vided into 365 equal parts, as was required.

I have often found this rule or method very useful in dividing circles into an odd number of equal parts, or wheels into odd numbers of equal siz'd teeth with equal spaces between them: and now I find it just as easy to divide any given circle into any odd number of equal parts, as to divide it into any even number. - And, for this purpose, I prefer the line of chords on a sector to that on a plain scale; because the sector may be opened so, as to make the radius of the line of chords upon it equal to the radius of the given circle, unless the radius of the circle exceeds the whole length of the sector when it is opened so as to resemble a straight ruler or scale; and this is what very seldom happens.

Any person who is used to handle the compasses, and the scale or sector, may very easily, by a little practice, take off degrees, and fractional parts of

42 SELECT EXERCISES.

of a degree, by the accuracy of his eye, from a line of chords, near enough the truth for the above-mentioned purpose.

Supposing the Distance between the Centers of two Wheels, one of which is to turn the other, be given; that the Number of Teeth in one of these Wheels is different from the Number of Teeth in the other, and it is required to make the Diameters of these Wheels in such Proportion to one another as their Numbers of Teeth are, so that the Teeth in both Wheels may be of equal Size, and the Spaces between them equal, that either of them may turn the other easily and freely: it is required to find their Diameters.

Here it is plain, that the distance between the centers of the wheels is equal to the sum of both their radii in the working parts of the teeth.—

Therefore,

Therefore, as the number of teeth in both wheels, taken together, is to the distance between their centers, taken in any kind of measure, as feet, inches, or parts of an inch ; so is the number of teeth in either of the wheels to the radius or semidiameter of that wheel, taken in the like measure, from its center to the working part of any one of its teeth.

Thus, suppose the two wheels must be of such sizes, as to have the distance between their centers 5 inches ; that one wheel is to have 75 teeth, and the other to have 33, and that the sizes of the teeth in both the wheels is equal, so that either of them may turn the other. The sum of the teeth in both wheels is 108 ; therefore say, As 108 teeth is to 5 inches, so is 75 teeth to $3\frac{47}{100}$ inches ; and as 108 is to 5, so is 33 to $1\frac{53}{100}$. So that, from the center of the wheel of 75 teeth to the working part of any tooth in it, is 3 inches and 47 hundred parts of an inch ; and, from

from the center of the wheel of 33 teeth to the working part of either of its teeth, is 1 inch and 53 hundred parts of an inch.

*The Description and Use of a New Machine
called the MECHANICAL PARADOX:*

The vulgar and illiterate take almost every thing, even the most important, upon the authority of others, without ever examining it themselves.—Although this implicit confidence is seldom attended with any bad consequences in the common affairs of life, it has nevertheless, in other things, been much abused; and in political and religious matters, has produced fatal effects. On the other hand, knowing and learned men, to avoid this weakness, have fallen into the contrary extreme: some of them believe every thing to be unreasonable, or impossible, that appears so to their first

first apprehension; not adverting to the narrow limits of the human understanding, and the infinite variety of objects, with their mutual operations, combinations, and affections, that may be presented to it.

It must be owned, that credulity has done much more mischief in the world than incredulity has done, or ever will do; because the influences of the latter extend only to such as have some share of education, or affect the reputation thereof.—And since the human mind is not necessarily impelled, without evidence, either to belief or unbelief; but may suspend its assent to, or dissent from, any proposition, till after a thorough examination; it is to be wished, that men of literature, especially philosophers, would not hastily, and by first appearances, determine themselves with respect to the truth or falsehood, possibility or impossibility of things.

A person who has made but little progress in the mathematics, though in other respects learned and judicious, would be apt to pronounce it impossible that two lines, which were no where two inches asunder, may continually approach toward one another, and yet never meet, although continued to infinity: and yet the truth of this proposition may be easily demonstrated.—And many, who are good mechanics, would be as apt to pronounce the same, if they were told, that although the teeth of one wheel should take equally deep into the teeth of three others, it should affect them in such a manner, that in turning it any way round its axis, it should turn one of them *the same way*, another *the contrary way*, and the third *no way at all*.

On a very particular occasion, about eighteen years ago, I contrived a small machine of this sort, which has been shewn

shewn and explained to many; and which I shall here describe, and explain some of the uses it has been applied to.

It is represented to view by Fig. 1. of Plate V. in which, *A* is called *the immoveable plate*, because it lies still on a table whilst the machine is at work. *BC* is a moveable frame, to be turned round an upright axis *a* (fixed into the center of the immoveable plate) by taking hold of the knob *n*, which is fixed into the index *b*.

On the said axis is fixed the immoveable wheel *D*, whose teeth take into the teeth of the thick moveable wheel *E*, and turn it round its own axis, as the frame is turned round the fixed axis of the immoveable wheel *D*; and in the same direction that the frame is moved.

The teeth of the thick wheel *E* take equally deep into the teeth of the three wheels *F*, *G*, and *H*; but operate on these wheels in such a manner, that whilst

whilst the frame is turned round, the wheel *H* turns *the same way* that the wheel *E* does; the wheel *G* turns *the contrary way*, and the wheel *F* turns *no way* at all.

Before we explain the principles on which these three different effects depend, it will not be improper to fix some certain *criteria* for bodies turning or not turning round their own axes or centers; and to make a distinction between absolute and relative motion.

1. If a body shews all its sides progressively round toward a certain fixed point in the heavens, the body turns round its own axis or center, whether it remains still in the same place, or has a progressive motion in any orbit whatever.—For, unless it does turn round its own center, it cannot possibly have one of its sides toward the west at one time, toward the south at another, toward the east at a third time, and toward the north at a fourth.—

This is the case with the Moon, which always keeps one side toward the earth; but shews the same side to every fixed point of the starry heaven in the plane of her orbit, in the time she goes once round her orbit; because in the time that she goes round her orbit, she turns once round her own axis or center.—On the contrary, if a body still keeps one of its sides toward a fixed point of the heaven, the body does not turn round its own axis or center, whether it keeps in one and the same place, or has a progressive motion in any orbit or direction whatever.—This is the case with the card of the compass in a ship, which still keeps one of its points toward the magnetic north, let the ship be at rest, or sail round a circle of many miles diameter.

Both these cases may be exemplified either by a cube or a globe, having a pin fixed into either of its sides to hold it by: we shall suppose a cube, because its sides are flat.—Sit down at a table,

and hold the cube by the pin, which may be called its axis, and keep one of its sides toward any side of the room. Whilst you do this, you do not turn the cube round its axis, whether you still keep it in the same place, or carry it round any other fixed body on the table.—But if you try to keep any side of the cube toward the fixed body, whilst you are carrying it round the same, you will find that you cannot do so, without turning the pin round (which is fixed into the cube) betwixt the finger and thumb whereby you hold it; unless you rise and walk round the table, keeping your face always toward the fixed body on the table; and then, both yourself and the cube will have turned once round; for the cube will have shewn the same side progressively round to all sides of the room, and your face will have been turned toward every side of the room, and every fixed point of the horizon.

2. If a ship turns round, and at the same time a man stands on the deck without moving his feet, he is turned absolutely round by the motion of the ship, though he has no relative motion with respect to the ship.—But if, whilst the ship is turning round, he endeavours to turn himself round the contrary way; he thereby only undoes the effect that the turning of the ship would otherwise have had upon himself; and is, in fact, so far from turning absolutely round, that he keeps himself from turning at all; and the ship turns round him, as round a fixed axis; although, with respect to the ship, he has a relative motion.

Fig. 2. is a small plan, or flat view of the machine, in which, the same letters of reference are put to the wheels in it, as to those in Fig. 1. for the conveniency of looking at both the figures, in reading the description of them. *WSEN* is the round immoveable plate: *D* the immoveable

wheel on the fixed axis in the center of that plate: *E* the thick moveable wheel, whose teeth take into the teeth of the wheel *D*; and *F* is one of the thin wheels, over which *G* and *H* may be put; and then, *F*, *G*, and *H* will make a thickness equal to the thickness of the wheel *E*, and its teeth will take equally deep into the teeth of them all. The frame that holds these wheels is represented by the parallelogram *abcd*; and if it be turned round, it can give no motion to the wheel *D*, because that wheel is fixed on an axis which is fixed into the great immovable plate.

Take away the thick wheel *E*, and leave the wheel *F* where it lies, on the lower plate of the frame. Then turn the frame round the axis of the immovable plate *WSEN* (denoted by *A* in Fig. 1.) and it will carry the wheel *F* round with it.—In doing this, *F* will still keep one and the same side toward the fixed central wheel *D*,
as

as the Moon still keeps the same side toward the Earth: and although *F* will then have no relative motion with respect to the moving frame, it will be absolutely turned round its own center *g* (like the man on the ship whilst he stood without moving his feet on the deck) for the cross mark on its side next *S* will be progressively turned toward all the sides of the room.

But, if we would keep the wheel *F* from turning round its own center, and so cause the cross mark upon it to keep always toward one side of the room; or, like the magnetic needle, to keep the same point still toward one fixed point in the horizon; we must produce an effect upon *F*, resembling what the man on the ship did, by endeavouring to turn himself round the contrary way to that which the ship turned, so as he might keep from turning at all; and by that means keep his face still toward one and the same point of the horizon.—And this

is done, by making the numbers of teeth equal in the wheels *D* and *F* (suppose 20 in each) and putting the thick wheel *E* between them, so as to take into the teeth of them both. For then, as the frame is turned round the axis of the fixed wheel *D*, by means of the knob *n*, the wheel *E* is turned round its axis by the wheel *D*; and, for every space of a tooth that the frame would turn the wheel *F*, in direction of the motion of the frame, the wheel *E* will counteract that motion, by turning the wheel *F* just as far backward with respect to the motion of the frame; and so will keep *F* from turning any way round its own center: and the cross mark near its edge will be always directed towards one side of the room.—Whether the wheel *E* has the same number of teeth as *D* and *F* have, or any different number, its effect on *F* will be still the same.

If *F* had one tooth less in number than *D* has, the effect produced on *F*, by the turning of the frame, would be

be as much more than counteracted by the intermediate wheel *E*, as is equal to the space of one tooth in *F*: and therefore, whilst the frame was turned once round, suppose in direction of the letters *WSEN* on the immoveable plate, the wheel *F* would be turned the contrary way, as much as is equal to the space taken up by one of its teeth. But, if *F* had one tooth more in number than *D* has, the effect of the motion of the frame (which is to turn *F* round in the same direction with it) would not be fully counteracted by means of the intermediate wheel *E*; for as much of that effect would remain as is equal to the space of one tooth in *F*: and therefore, in the time the frame was turned once round, the wheel *F* would turn, on its own center, in direction of the motion of the frame, as much as is equal to the space taken up by one of its teeth: and here note, that the wheel *E* (which turns *F*) always turns in direction of the motion of the frame.

And therefore, if an upright pin be fixed into the lower plate of the frame, under the center of the wheel *F*, and if the wheel *F* has the same number of teeth that the fixed wheel *D* has, the wheel *G* one tooth less, and the wheel *H* one tooth more; and if these three wheels are put loosely upon this pin, so as to be at liberty to turn either way; and the thick wheel *E* takes into the teeth of them all, and also into the teeth of the fixed wheel *D*; then, whichever way the frame is turned, the wheel *H* will turn *the same way*, the wheel *G* *the contrary way*, and the wheel *F* *no way* at all.—The less number of teeth *G* has, with respect to those of *D*, the faster it will turn backward; and the greater number of teeth *H* has, with respect to those in *D*, the faster it will turn forward; reckoning *that* motion to be backward which is contrary both to the motion of the frame and of the thick wheel *E*, and *that* motion to be forward which is in the same direction with the motion of the

the frame and of the wheel *E*.—So that the turning or not turning of the three wheels, *F*, *G*, *H*, or the direction and velocity of the motions of those that do turn round, depends entirely on the relation between their numbers of teeth and the number of teeth in the fixed wheel *D*, without any regard to the number of teeth in the moveable wheel *E*.

Having solved the paradox, and described the cause of the different effects which are produced upon the three wheels F, G, and H, we shall now proceed to shew some uses that may be made of the machine.

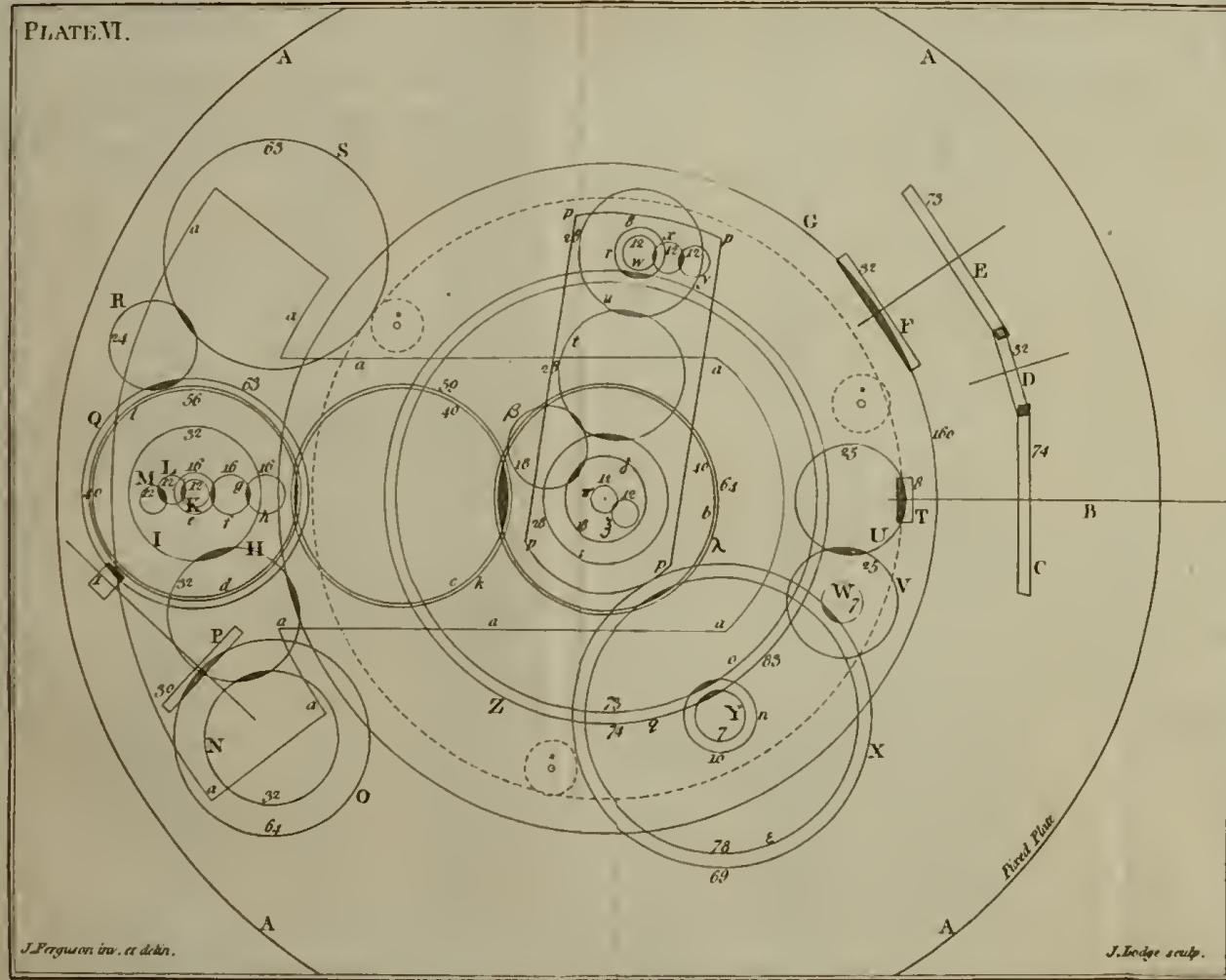
This machine is so much of an ORRERY, as is sufficient to shew the different lengths of days and nights, the vicissitudes of the seasons, the retrograde motion of the nodes of the Moon's orbit, the direct motion of the apogeeal point of her orbit, and the months in which the Sun and Moon must be eclipsed.

On

On the great immoveable plate *A* (see Fig. 1.) are the months and days of the year, and the signs and degrees of the zodiac so placed, that when the annual index *b* is brought to any given day of the year, it will point to the degree of the sign in which the Sun is on that day.—This index is fixed to the moveable frame *BC*, and is carried round the immoveable plate with it, by means of the knob *n*. The carrying this frame and index round the immoveable plate, answers to the Earth's annual motion round the Sun, and to the Sun's apparent motion round the ecliptic in a year.

The central wheel *D* (being fixed on the axis *a*, which is fixed in the center of the immoveable plate) turns the thick wheel *E* round its own axis by the motion of the frame; and the teeth of the wheel *E* take into the teeth of the three wheels *F*, *G*, *H*, whose axes turn within one another, like the axes of the hour, minute, and second hands

PLATE VI.



J. Ferguson inv. et delin.

J. Lodge sculp.

hands of a clock or watch, where the seconds are shewn from the center of the dial-plate.

On the upper ends of these axes are the round plates *I*, *K*, *L*; the plate *I* being on the axis of the wheel *F*; *K* on the axis of *G*, and *L* on the axis of *H*. So that, whichever way these wheels are affected, their respective plates, and what they support, must be affected in the same manner; each wheel and plate being independent of the others.

The two upright wires *M* and *N* are fixed into the plate *I*; and they support the small ecliptic *OP*, on which, in the machine, the signs and degrees of the ecliptic are marked.—This plate also supports the small terrestrial globe *e* on its inclining axis *f*, which is fixed into the plate near the foot of the wire *N*. This axis inclines $23\frac{1}{2}$ degrees from a right line, supposed to be perpendicular to the surface of the plate *I*, and $66\frac{1}{2}$ to the plane of the small ecliptic

ecliptic OP which is parallel to that plate.

On the Earth e is the crescent g , which goes more than half way round the Earth, and stands perpendicular to the plane of the small ecliptic OP , directly facing the Sun Z : its use is to divide the enlightened half of the Earth next the Sun from the other half which is then in the dark; so that it represents the boundary of light and darkness, and therefore ought to go quite round the Earth; but cannot, in a machine, because, in some positions, the Earth's axis would fall upon it.—The Earth may be freely turned round on its axis by hand, within the crescent, which is supported by the crooked wire w , fixed to it, and into the upper plate of the moveable frame BC .

In the plate K are fixed the two upright wires Q and R : they support the Moon's inclined orbit ST in its nodes, which are the two opposite points of the

the Moon's orbit where it intersects the ecliptic OP . The ascending node is marked α , to which the descending node is opposite, below e , but hid from view by the globe e . The half αTe of this orbit is on the north side of the ecliptic OP , and the other half $eS\alpha$ is on the south side of the ecliptic. The Moon is not in this machine: but, when she is in either of the nodes of her orbit in the heavens, she is then in the plane of the ecliptic: when she is at T in her orbit, she is in her greatest north latitude; and when she is at S , she is in her greatest south latitude.

In the plate L is fixed the crooked wire UU , which points downward to the small ecliptic OP , and shews the motion of the Moon's apogee therein, and its place at any given time.

The ball Z represents the Sun, which is supported by the crooked wire XY , fixed into the upper plate of the frame at X . A straight wire W proceeds from the Sun Z , and points always toward the

the center of the Earth e ; but toward different points of its surface at different times of the year, on account of the obliquity of its axis, which keeps its parallelism during the Earth's annual course round the Sun Z ; and therefore must incline sometimes toward the Sun, at other times from him, and twice in the year neither toward nor from the Sun, but sidewise to him. The wire W is called *the solar ray*.

As the annual index b shews the Sun's place in the ecliptic for every day of the year, by turning the frame round the axis of the immoveable plate A , according to the order of the months and signs, the solar ray does the same in the small ecliptic OP : for, as this ecliptic has no motion on its axis, its signs and degrees still keep parallel to those on the immoveable plate. At the same time, the nodes of the Moon's orbit ST (or points where it intersects the ecliptic OP) are moved backward,

backward, or contrary to the order of signs, at the rate of $19\frac{1}{2}$ degrees every Julian year; and the Moon's apogeal wire *UU* is moved forward, or according to the order of the signs of the ecliptic, nearly at the rate of 41 degrees every Julian year; the year being denoted by a revolution of the Earth *e* round the Sun *Z*; in which time the annual index *b* goes round the circles of months and signs on the immovable plate *A*.

Take hold of the knob *n*, and turn the frame round thereby; and in doing this, you will perceive that the north pole of the Earth *e* is constantly before the crescent *g*, in the enlightened part of the Earth toward the Sun, from the 20th of March to the 23d of September; and the south pole all that time behind the crescent in the dark: and, from the 23d of September to the 20th of March, the north pole is constantly in the dark, behind the crescent, and the south pole in the light before it: which

which shews, that there is but one day and one night at each pole, in the whole year; and that, when it is day at either pole, it is night at the other.

From the 20th of March to the 23d of September, the days are longer than the nights in all those places of the northern hemisphere of the Earth which revolve through the light and dark, and shorter in those of the southern hemisphere.—From the 23d of September to the 20th of March, the reverse.

There are 24 meridian semicircles drawn on the globe, all meeting in its poles; and as one rotation or turn of the Earth on its axis is performed in 24 hours, each of these meridians is an hour distant from the other, in every parallel of latitude.—Therefore, if you bring the annual index *b* to any given day of the year, on the immoveable plate, you may see how long the day then is at any place of the Earth, by counting how many of these meridians

meridians are in the light, or before the crescent, in the parallel of latitude of that place; and this number being subtracted from 24 hours, will leave remaining the length of the night.— And if you turn the Earth round its axis, all those places will pass directly under the point of the solar ray, which the Sun passes vertically over on that day, because they are just as many degrees north or south of the equator, as the Sun's declination is then from the equinoctial.

At the two equinoxes, *viz.* on the 20th of March and 23d of September, the Sun is in the equinoctial, and consequently has no declination. On these days, the solar ray points directly toward the equator, the Earth's poles lie under the inner edge of the crescent, or boundary of light and darkness; and, in every parallel of latitude, there are twelve of the meridians, or hour-circles, before the crescent, and twelve behind it; which shews that the days

and nights then are each twelve hours long at all places of the Earth. And, if the Earth be turned round its axis, you will see that all places on it go equally through the light and the dark hemispheres.

On the 21st of June, the whole space within the north polar circle is enlightened, which is $23\frac{1}{2}$ degrees from the pole, all around; because the Earth's axis then inclines $23\frac{1}{2}$ degrees toward the Sun; but the whole space within the south polar circle is in the dark; and the solar ray points toward the tropic of Cancer on the Earth, which is $23\frac{1}{2}$ degrees north from the equator.—On the 20th of December the reverse happens, and the solar ray points toward the tropic of Capricorn, which is $23\frac{1}{2}$ degrees south from the equator.

If you bring the annual index *b* to the beginning of January, and turn the Moon's orbit *ST* by its supporting wires *Q* and *R* till the ascending node (marked

(marked α) comes to its place in the ecliptic OP , as found by an Ephemeris, or by Astronomical Tables, for the beginning of any given year; and then move the annual index by means of the knob n , till the index comes to any given day of the year afterward, the nodes will stand against their places in the ecliptic on that day.— And if you move the index onward, till either of the nodes comes directly against the point of the solar ray, the index will then be at the day of the year on which the Sun is in conjunction with that node. At the times of those new Moons which happen within seventeen days of the conjunction of the Sun with either of the nodes, the Sun will be eclipsed: and at the times of those full Moons, which happen within twelve days of either of these conjunctions, the Moon will be eclipsed.— Without these limits there can be no eclipse either of the Sun or Moon; because, in nature, the

Moon's latitude, or declination from the ecliptic, is too great for the Moon's shadow to fall on any part of the Earth, or for the Earth's shadow to touch the Moon.

Bring the annual index to the beginning of January, and set the Moon's apogeal wire *U U* to its place in the ecliptic for that time, as found by Astronomical Tables: then move the index forward to any given day of the year, and the wire will point on the small ecliptic to the place of the Moon's apogee for that time.

The Earth's axis *f* inclines always toward the beginning of the sign Cancer on the small ecliptic *O P*.—And, if you set either of the Moon's nodes, and her apogeal wire, to the beginning of that sign, and turn the plate *A* about, until the Earth's axis inclines toward any side of the room (suppose the north side) and then move the annual index round and round the immoveable plate *A*, according to the order

order of the months and signs upon it, you will see that the Earth's axis and beginning of Cancer will still keep toward the same side of the room, without the least deviation from it; but the nodes of the Moon's orbit ST will turn progressively towards all the sides of the room, contrary to the order of signs in the small ecliptic OP , or from east, by south, to west, and so on: and the apogeal wire UU will move the contrary way to the motion of the nodes, or according to the order of the signs in the small ecliptic, from west, by south, to east, and so on quite round.—A clear proof that the wheel F , which governs the Earth's axis and the small ecliptic, does not turn any way round its own center; that the wheel G , which governs the Moon's orbit OP , turns round its own center backward, or contrary both to the motion of the frame BC and thick wheel E ; and that the wheel H , which governs the Moon's apogeal wire UU ,

turns round its own center, forward, or in direction both of the motion of the frame, and of the thick wheel *E*, by which the three wheels, *F*, *G*, and *H*, are affected.

The wheels *D*, *E*, and *F*, have each 39 teeth in the machine; the wheel *G* has 37, and *H* 44; as shewn in Fig. 3.

The parallelism of the Earth's axis is perfect in this machine; the motion of the apogee very nearly so; the motion of the nodes not quite so near the truth, though they will not vary sensibly therefrom in one year.—But they cannot be brought nearer, unless larger wheels, with higher numbers of teeth, are used.

In nature, the Moon's apogee goes quite round the ecliptic in eight years and 312 days, in direction of the Earth's annual motion; and the nodes go round the ecliptic, in a contrary direction, in eighteen years and 225 days.—In the machine, the apogee goes round the ecliptic *OP* in eight years.

years and four-fifths of a year, and the nodes in eighteen years and a half.

Notwithstanding the difference of the numbers of teeth in the wheels *F*, *G*, and *H*, and their being all of equal diameters, they take tolerably well into the teeth of the thick wheel *E*, because they are made of soft wood. —But, if they were made of metal, the wheel *E* in Fig. 1. ought to be made of the shape of *E* (seen edge-wise) in Fig. 3. with very deep teeth: and the wheels *F*, *G*, and *H*, in Fig. 1. of diameters proportioned to their respective numbers of teeth, as *F*, *G*, and *H*, in Fig. 3. And then the teeth of these three wheels would be of equal sizes with those of the wheel *E* wherein they work: and the motions would be free and easy, without any pinching or shake in the teeth.

An Orrery, shewing the Motions of the Sun, Mercury, Venus, Earth, Moon, and Nodes of the Moon's Orbit; the different Lengths of Days and Nights, the Vicissitudes of Seasons, Age and Phases of the Moon, and all the Solar and Lunar Eclipses.

The use of this Orrery, and the manner of using it, being already described in my Astronomy, I shall not repeat those matters here; but shall only describe the wheel-work of it, which is not done in that book. It is not copied from any other Orrery whatever, and I can with truth say, that it shews the revolutions of the Moon and Planets nearer the truth than any other Orrery does, that has fallen under my examination. I therefore freely give the following account of it to the Public, in the best manner that I can; and do wish the description may be generally understood. To any

any Clock-maker I hope it will be plain, and to every Orrery-maker I believe it will be quite so.

The fifth Plate is a plan of the wheel-work, in which the diameter of each wheel is equal to the semidiameter or radius thereof in the Orrery I have made: the numeral figures at each wheel shew the number of teeth in that wheel, and the shaded parts shew where the teeth of any one wheel takes into the teeth of another, as the one turns the other.

The sixth Plate is a section or side-view of all the wheel-work that could be brought into sight. But in this, some few wheels could not be shewn; for, in the Orrery itself, take a view of the wheels on any side you please, some of them will be unavoidably hid from sight by others that are between them and the eye.

Those in Plate VII. that come in sight have the same numeral figures set to them as the like ones have in Plate VI.

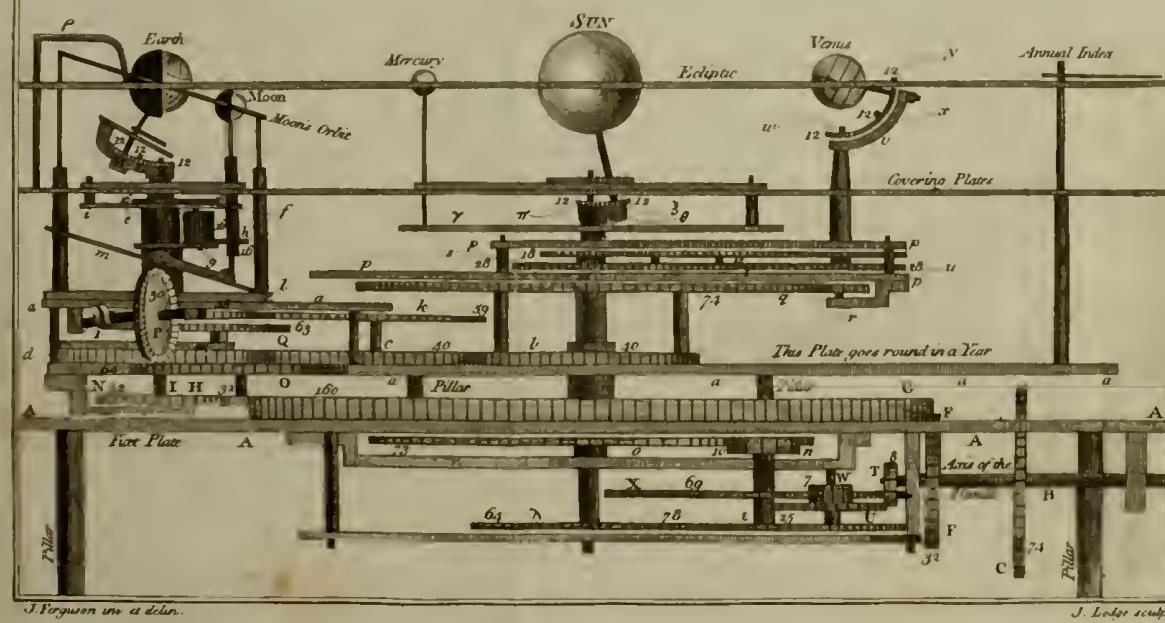
and

and also the same letters of reference where there is room to insert them. And therefore, in reading the description of Plate VI. it will be requisite to look first at it, and then at Plate VII. ; by which means the Reader will see the position of these wheels with respect to each other, as they are placed higher or lower in the frames which contain them.

AAAA is a round immovable plate supported by four pillars ; some of the wheels are below it, but the greatest number of them are above it. It supports and bears the weight of them all.

B is the axis of the handle or winch by which all the wheels are turned : on its axis is a wheel *C* of 74 teeth, which turns a wheel *D* of 32, and *D* turns a wheel *E* of 73 teeth, on whose axis is a wheel *F* of 32, turning a wheel *G* of 160 teeth, which turns a wheel *H* of 32, and *H* turns a wheel *I* of the same number, on the top of whose axis is

PLATE VII.



is a small wheel *K* of 12 teeth (just under the Earth) which turns a wheel *L* of the same number and size; and *L* turns such another wheel *M* of the same number. The axis of *M* inclines $23\frac{1}{2}$ degrees, and the Earth on the top of it is turned round by it. The wheel *H* of 32 teeth turns a wheel *N* of the same number, on the top of whose axis is an index which goes round a circle of 24 hours, (on the plate that covers the wheel-work) in the time the Earth turns round its axis.—The wheels *D* and *E* could not be shewn in Plate VII. because the wheel *C* of 74 teeth hides them from sight.

On the axis of the wheel *N* is a wheel *O* of 64 teeth, turning a concentrate wheel *P* of 30, on whose axis is an endless screw, of a single thread 1, turning a wheel *Q* of 63 teeth, which carries the Moon round the Earth in her orbit, from change to change, in 29 days 12 hours 45 minutes. This wheel of 63 teeth turns a wheel *R* of

24, which turns a wheel *S* of 63 teeth round in 29 days 12 hours 45 minutes, on whose axis is an index that shews the days of the Moon's age on a circle of $29\frac{1}{2}$ equal parts, on the plate that covers the wheel-work.

On *B*, the axis of the handle, is a pinion *T* of 8 leaves, turning a wheel *U* of 25 teeth, which turns another wheel *V* of the same number and size, on whose axis is a pinion *W* of 7 leaves, turning a wheel *X* of 69 teeth, on whose axis is a pinion *Y* of 7 leaves, turning a wheel *Z* of 83 teeth once round in 365 days 5 hours 48 minutes 57 seconds, and carrying the Earth round the Sun in that time. For, in this wheel are four short pillars, whose upper ends are fixed into the lower plate of a moveable frame *aaaaaaa* (Plate VI.) that turns round on a fixed upright pin in the center of the plate *AAAA*, and contains the above-mentioned wheels belonging to the Earth and Moon: so that the whole frame goes

goes round the center pin in the same time with the wheel *Z*.

This last wheel cannot be seen in Plate VII. because it lies within the wheel *G*, which is only a thick ring having 160 teeth on its outside. Its innermost side is represented by a dotted circle in Plate VI. and it is kept in its place by three rollers, marked *** , which turn upon pins fixed in the great immoveable plate *AAAAA*.

As the uppermost edge of the con-
trate wheel *F* (see Plate VII.) must
come a little way through the plate
AAAAA, in order to turn the ring-
wheel *G* that lies on the upper side of
this plate, and this wheel turns the
wheel *H* of 32 teeth that belongs to the
Earth's diurnal motion; it is plain,
that as the wheel *H* must go round *G*
in a year by the annual part of the
work, *G* must be thick enough to turn
H at such a distance from or above the
plate *AAAAA*, that *H* may go over the
top of *F* without touching it: other-

wise, when *H* came round to *F*, it could not pass by, but would stop the annual motion.

In the center, just above the upper surface of the moveable frame-plate *aaaa*, is a fixed wheel *b* of 40 teeth taking into the teeth of the wheel *c*, which is also 40 in number; and these take into the teeth of a wheel *d*, whose number is 40 also. The axis of this last wheel is hollow, and the top of it is fixed tight at *K* (see Plate VII.) in the piece *KLM* that carries the Earth.—This part of the work keeps the Earth's inclined axis in a constant parallelism in its annual course round the Sun. For, as *d* is connected with the fixed wheel *b*, by means of the intermediate wheel *c*, and *c* rolls or goes round *b* by the annual work, and as *b*, *c*, and *d* have equal numbers of teeth, *d* must always preserve its parallelism throughout its annual motion. The axis of *b* is fixed into the immovable plate *AAAA*; and it is hollow, to let the

the axes of some wheels below that plate turn within it.

The solid spindle, or axis of the wheel *I* of 32 teeth, turns within the hollow axis of the wheel *d* of 40; and on the top of this solid spindle is the small wheel *K* of 12 teeth, which turns the Earth round its axis by the wheels *L* and *M*, of equal number and size with *K*, as already mentioned.

The hollow axis of the parallelism-wheel *d* is within an upright socket, whose lowermost end is fixed into the top-plate (marked 56 in Plate VII.) of the moveable frame *aaaa*, and on the top of this socket is fixed a small wheel *e* of 16 teeth, which take into the teeth of another wheel *f* of the same number and size; on the axis of which is a long pinion *g* of 16 leaves, which take into the wheel *b* of 16 teeth, whose axis is hollow, and has a black cap on the top of it, covering just one half of the Moon.—Now, as the socket, on whose top the wheel *e* is placed,

placed, is fixed into the annual moving frame, it is plain, that, whichever side or tooth of the wheel *e* is once toward the Sun, will always be so; and therefore, as the wheel *f*, the pinion *g*, and the wheel *h*, go all round the wheel *e* by the work that carries the Moon round the Earth, and all these have equal numbers of teeth, the wheel *h* will always keep the Moon's cap facing toward the Sun, and shew her to be always full as seen from the Sun, but continually changing her phases as seen from the Earth in her going round it. For, when the Moon is between the Earth and the Sun (as represented in Plate VII.) her cap will hide the the whole of her from the Earth: but, when she is opposite to the Sun, all the half or side of her next the Earth will then appear like a full Moon, before the circular edge of the cap: and when she is mid-way between these positions, or in either of her quadratures, she will appear just half enlightened as seen from the Earth.

The

The axis of the wheel \mathcal{Q} , of 63 teeth, which carries the Moon round the Earth, is hollow, and turns round upon the above-mentioned fixed socket. To the top of this axis (just under the wheel e of 16 teeth, Plate VII.) is fixed the bar if , which carries the Moon round the Earth by the motion of the wheel \mathcal{Q} .

On the top of the axis of the wheel c of 40 teeth, is a wheel k of 59, turning a wheel l of 56, which cause the nodes of the Moon's orbit to go once round, with a retrograde motion, thro' all the signs and degrees of the ecliptic, in $18\frac{2}{3}$ years. The axis of l is hollow, and turns upon the hollow axis of \mathcal{Q} ; and on the axis of l is a circular plate m (Plate VII.) fixed obliquely on that axis, and parallel to the Moon's orbit. The work that carries the Moon round the Earth carries also the piece g round upon this oblique plate; and, as the lower end of the Moon's axis (which turns within the hollow axis

of her cap) is fixed into the piece *g*, it causes the Moon to rise and fall in her oblique orbit, according to her north or south latitude or declination from the ecliptic. As the nodes of her orbit are even with the plane of the ecliptic, one half of her orbit is on the north side, and the other half on the south side of the ecliptic.

On the axis of the wheel *X*, which has 69 teeth, is a pinion *n* of 10 leaves, turning a wheel *o* of 73 teeth, which carries Venus round about the Sun in 224 days 17 hours. The axis of the wheel *o* is hollow (because another axis turns within it) and on the top of it is fixed the lower plate of the frame *pppp*, which carries Venus round the Sun, and has wheels within it belonging to Venus and to Mercury.

Under the lowest plate of this frame is a fixed wheel *q* of 74 teeth, of the same diameter as the wheel *Z* of 83, which gives the Earth its annual motion; so that, in Plate VI. one and the same

same circle represents both the wheels. —A pinion *r* of 8 leaves takes into the teeth of the fixed wheel *q*, and is carried round *q* by the motion of the frame *pppp*, that carries Venus round the Sun. Consequently, in the time this pinion is carried round the wheel, it will turn $9\frac{1}{4}$ times round its axis, equal to the number of Venus's days and nights in the time she goes round the Sun.

The wheel *q* of 74 teeth is fixed on the same (above-mentioned) socket on which the wheel *b* of 40 teeth is fixed. The top of this socket goes through the lower plate of the frame *pppp*, and a wheel *s* of 28 teeth is fixed upon the top of this socket, just above the same plate. Another wheel *t* of 28 teeth takes into the teeth of *s*, and is carried round it by the motion of the frame: and a third wheel *u* of 28 teeth (which is also carried round by the frame) takes into the teeth of *t*: the axis of *u* is hollow, it turns upon

the solid spindle or axis of the pinion r of 8 leaves, and on its top is fixed the curved piece v (Plate VII.) that carries Venus on her inclined axis, which, by means of the three last-mentioned wheels of 28 teeth, is kept in a constant parallelism in going round the Sun.

On the top of the axis of the pinion r of 8 leaves, and just above the curved piece v (Plate VII.) is a small wheel w of 12 teeth, which turns another wheel x of the same number and size; and this last wheel turns a third wheel y of the same number, which is fixed on the axis of Venus, and turns her $9\frac{1}{4}$ times round her axis in the time she goes round the Sun; which is just as often as the pinion r turns round in the time it is carried round the fixed wheel q of 74 teeth.

On the top of the axis of the middle wheel t of 28 teeth, is another wheel of the same number and size, which turns a wheel β of 18 teeth; and this

wheel turns another wheel δ of the same number, whose axis is a hollow socket, on which a bar γ (Plate VII.) is fixed; and this bar carries Mercury round the Sun in 87 days 23 hours.

On the axis of the wheel X (already mentioned) of 69 teeth is a wheel ϵ of 78, which turns a wheel λ of 64 round its axis in 25 days 6 hours. The axis of this wheel turns within the above-mentioned hollow arbors in the center; and on its top is the small wheel π of 12 teeth, which turns another wheel ξ of the same number and size: this last wheel is fixed on the Sun's axis, it turns in the fixed piece θ (Plate VII.) and turns the Sun round his axis in 25 days 6 hours.

The Sun's axis inclines $7\frac{1}{2}$ degrees from a perpendicular to the ecliptic; Venus's axis 75 degrees, and the Earth's axis $23\frac{1}{2}$.

The Earth turns round within a black cap, that always covers the half of it which at any instant of time is

turned quite away from the Sun: the edge of the cap represents the *solar horizon*, or circle bounding light and darkness: it is supported by a crooked wire ρ , whose lower end is fixed into the plate that covers the wheels, and is carried round by the annual motion-work. An index (called the *annual index*) goes round the ecliptic, by the same work, keeping always opposite to the Sun, and shewing the days of the months, and the Sun's apparent place in the ecliptic as seen from the Earth.

On looking at Plate VI. it may perhaps appear, even to a very ingenious mechanic, that the wheels C , D , and E are superfluous; and that the wheel F , which gives motion to the toothed ring G , might have been upon the axis B of the handle. For, as F has 32 teeth, and H , that is turned by the teeth of G , (and turns the Earth round its axis) has also 32 teeth, F and H would turn round in equal times; and confe-

consequently, a turn of the handle would have answered to a turn of the Earth on its axis.—This indeed would have been the case if the Earth had no annual motion: but as *H* goes round *G* in a year, the same way that *G* turns round, *H* loses five turns in going round *G* (for 5 times 32 is 160, the number of teeth in *G*), and then the handle would have turned 370 times round in the time the Earth made 365 rotations.—To prevent this, and so make the turns of the Earth and handle agree together, *C* has 74 teeth, and *E* only 73. So that the wheel *E* will turn 5 times oftener round than the handle does in 365 turns thereof; and consequently make the Earth's daily rotation equal to a turn of the handle, or to 24 hours of mean solar time.

Another Orrery.

This is the Orrery mentioned in my *Tables and Tracts* (page 169, 2d edition) which I intended to keep for my son, who was then serving an apprenticeship to a mathematical instrument-maker. But, as it has pleased God to call him from this world to a better, I shall now freely communicate it to the Public.

It shews the length of day and night at all places of the Earth, every day of the year, with the Sun's true place, declination, time of rising and setting, the hour of the day, the Sun's altitude, azimuth, and the variation of the compass at any place. Also the Moon's periodical and synodical revolution, her motion on her axis, her latitude, altitude, azimuth, rising and setting; her mean anomaly and elliptic equation; with the days of all the new and full Moons and eclipses, for 6000 years before

before and after the Christian æra.— The outside figure of this Orrery is exactly shewn in the eighth plate of my Astronomy; but the inside work differs much from what is represented in the second figure of that plate: and this inside work is what I shall now describe.

A large wheel of 235 teeth is fixed in the box that contains the work, the center of the wheel being in the center of the box, directly under the Sun's center. On this wheel runs a pinion of 19 leaves, carried round the teeth of the wheel by the annual motion of the Earth; and by this means, the pinion is turned round its own axis for every 19 teeth that it is carried onward, in going round the wheel.— Now, supposing this pinion to be carried round the wheel in $365\frac{1}{4}$ days, the pinion will be turned round its own axis in 29 days 12 hours 44 minutes 25 seconds, and a bar on the axis of the pinion will carry the Moon round the

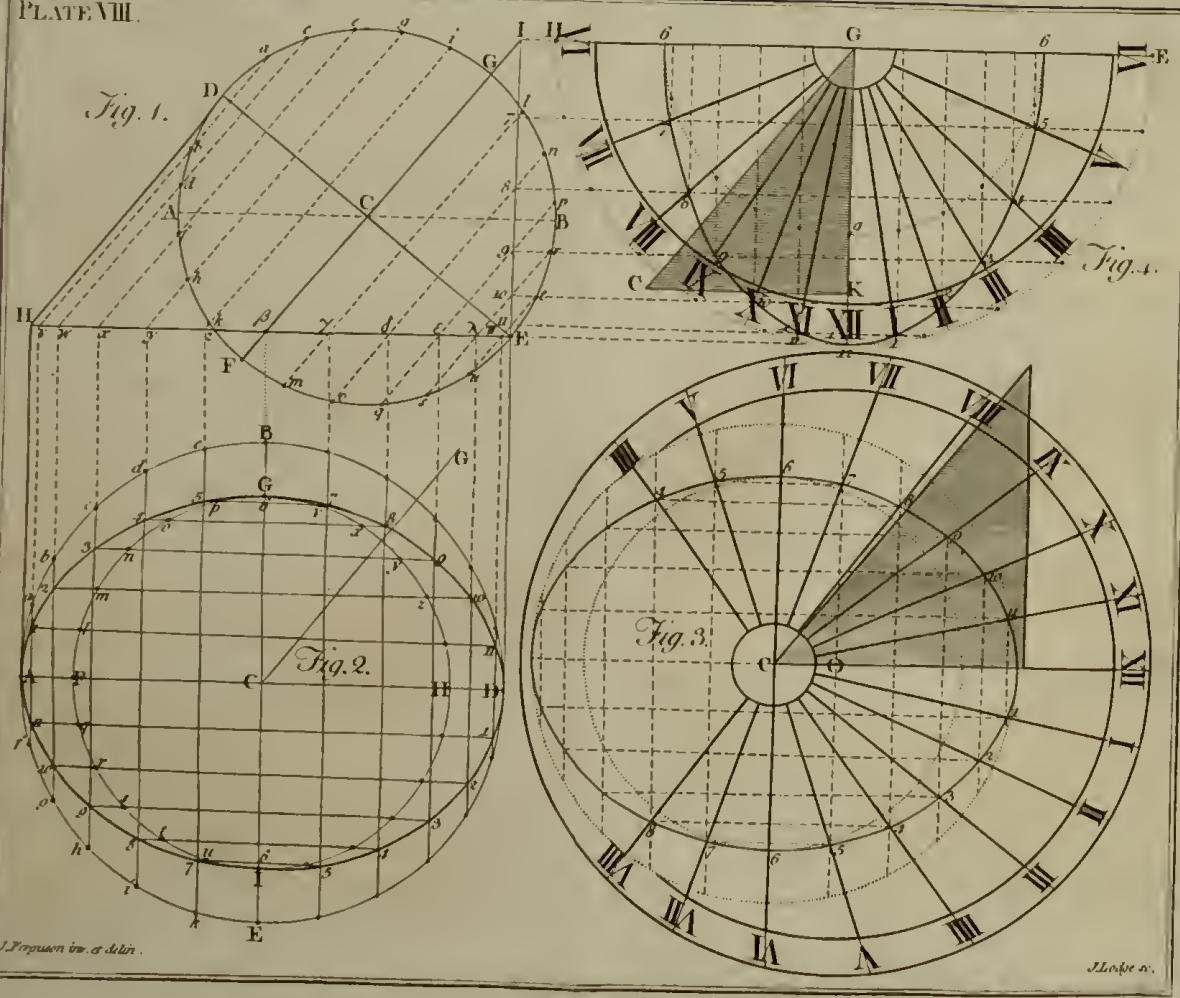
the Earth, from change to change, in that time. This comes so near to the truth, as to vary but one day in the Moon's course in 335 years; and these are the nearest numbers for such simple wheel-work that can possibly be found for mean lunations.

But, in nature, the Earth moves unequally round the Sun, so that there are 8 days more between the vernal and autumnal equinox, than between the autumnal and vernal.—And therefore, in common Orreries, where this circumstance is taken no notice of, the Earth's position to the Sun cannot be right at both the equinoctial points.

In order to avoid this error, I first divided the ecliptic into 360 equal parts for degrees; and then, after having put the names of the signs to it, I laid down the days of the year from an ephemeris against the degrees of the Sun's place in the ecliptic, for each day respectively throughout the year. By this means, the daily spaces,

answer-

PLATE VIII.



answering to the Earth's unequal motion round the Sun, were so divided, as to be continually and gradually lessening from the 30th of December till the first of July; and then as gradually lengthening from the first of July till the 30th of December; as the Earth's progressive annual motion is swiftest of all on the 30th of December, and slowest of all on the first of July.

The days of the months being unequally divided, so as to answer to the Earth's unequal motion round the Sun, I made these divisions a pattern or scale for dividing the 235 teeth of the above-mentioned wheel into such unequal spaces as would agree with the spaces allotted for the days answering to them. But these gradual inequalities of the teeth were so very small, and the difference so little between the widest and narrowest, that the pinion (whose leaves were all equal) run very smoothly through all the teeth of the wheel; as the leaves of

of the pinion were sized to these teeth, which were at a mean rate between the greatest and least distances from one another. By this contrivance, the mean lunation was always 29 days 12 hours 44 minutes 25 seconds throughout the whole year; and the pinion was among the least distant teeth of the wheel when the annual index was at the first of July, and among the most distant teeth when the index was at the 30th of December.

For the parallelism of the Earth's axis, a wheel of 59 teeth was fixed in the middle of the work, with its center directly over the center of the wheel of 235 teeth, and the teeth of another wheel of 59 took into the teeth of the former; and those into the teeth of a third wheel of the same number, on the top of whose axis the piece that carries the Earth on its oblique axis was fixed. And, as the Earth was moved round the Sun, these three wheels kept the parallelism of the Earth's axis, as described in the *Mechanical*

chanical Paradox, and the former Orrery.

Above the middlemost wheel of 59, and on its axis, is fixed a wheel of the same number, which takes into a wheel of 56 teeth: this last wheel is just below the Earth, and turns the nodes of the Moon's orbit quite round backward, in $18\frac{2}{3}$ years.

Above the last-mentioned wheel of 59 teeth, and on the same axis with it, is a wheel of 55, turning a wheel of 62 teeth below the Earth; and this wheel of 62 moves the Moon's apogee plate quite round forward, in 8 years 312 days. And from this plate a wire rises, and points out the place and motion of the apogee in the Moon's ecliptic.

[*By comparing this description with that of the Orrery in the 8th plate of my Astronomy (see § 399.), it will be very easily understood: particularly those parts which shew the parallelism of the Earth's axis, the motion of the Moon's nodes, and apogee.*]

As

As the Moon goes round the Earth, she comes to her mean changes, nodes, and apogee, in the proper times; and, at all intermediate times, her distance from her apogee and nodes are shewn in her ecliptic, orbit, and apogee plate; on which last her mean anomaly and elliptic equation are shewn: by which means her true place in the ecliptic, and her latitude, may be very nearly found for any given time.

The days of the months, throughout the year, are laid down in a diagonal manner, in a spiral line of four revolutions, marked 0, 1, 2, 3, for leap year, and the first, second, and third years after. The annual index, in these spirals, being at the given day of any month, for either of these years, all the other motions and phenomena will be right for that day: and, by means of these diagonals, the lunation is brought still nearer the truth than as above specified.

Within this set of spirals are tables, which shew the places of the Sun, Moon,

Moon, Ascending Node, and Apogee, for the noon of the first day of January, in any year within the limits of 6000 years both before or after the Christian æra. And, by means of these tables, the Orrery may, in less than two minutes of time, be rectified for the beginning of any of these years; and then, all the motions, not only for that year, will be right, but also for 334 years afterward, without needing any rectification.

A New Geometrical Method of constructing Sun-Dials.

Draw at pleasure the horizontal line ACB (Plate VIII. Fig. 1.) and on the point C , as a center, describe the circle $DGEF$. Draw the diameter DCE , so as to make an angle (DCA) with ACB equal to the co-latitude of the place for which the dial is to serve; and draw FCG at right angles (or perpendicular) to ACB .

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pendicular) to DCE : then ACB shall represent the horizon of the place, DCE the equinoctial, ECG the axis of the world and stile of the dial, G the north pole, F the south pole, and the arc BG the elevation of the pole above the north point of the horizon; which elevation is equal to the latitude of the place.

From the point E draw the right line EH parallel to the horizon ACB , and from the point D draw DH parallel to CF . So EH shall be equal to the longest diameter of an ellipsis (Fig. 2.) and DE equal to the shortest diameter thereof. Divide the circle $DGEF$ into 24 equal parts, beginning at D ; and connect the division-points which are equidistant from D by the straight lines $ab, cd, ef, \&c.$ continuing these lines down to the points v, w, x, y, z , in that part of the line HE that falls without the circle.

From the point β , where GCF intersects HE , draw βBCE (Fig. 2.) perpendi-

perpendicular to $H\beta E$ (Fig. 1.) and draw ACD in Fig. 2. parallel to $H\beta E$ in Fig. 1. So, in Fig. 2. BCE and ACD shall cross each other at right angles in the point C .—On this point, as a center, with the length βH or βE in Fig. 1. as a radius, describe the circle $ABDE$ in Fig. 2. and divide it into 24 equal parts, beginning at A , and connect the division-points, which are equidistant from A , by the straight lines $af, bg, ch, di, ek, \&c.$ Then, from Fig. 1. take CD in your compasses, as a radius; and, with that extent, on C as a center in Fig. 2. describe the circle $FGHI$, and divide it into 24 equal parts, beginning at G . Through these division-points $p, v, o, x, n, y, \&c.$ which are equidistant from G , draw the right lines $57, 48, 39, 210, \&c.$ meeting the lines within the former circle at the points $5, 7, 4, 8, 3, 9, 2, 10, 1, 11$, on the side AGD ; and at $7, 5, 8, 4, 9, 3, 10, 2, 11, 1$, on the side AID . Then, through these points of meeting, draw by a steady hand the ellipsis

$A \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$, &c. whose longest diameter AD is equal to $H\beta E$ in Fig. 1. and its shortest diameter GI equal to DCE in the same figure, as above mentioned.

This done (which may be much sooner done than described) lay the edge of a straight ruler to the center C in Fig. 2. and to the above-mentioned division-points 5, 7, 4, 8, &c. in the ellipses, and draw straight lines from C through these points, as in Fig. 3.; and they will be the true hour-lines on a horizontal dial.

Lastly, from the center C , in Fig. 2. draw the straight line CG parallel to CG in Fig. 1. for the axis of the stile, or edge thereof that casts a shadow on the time of the day; and CG shall be parallel to the axis of the world when the dial is truly set, as the like edge of the stile of every dial must be.

[*N. B.* Straight lines, parallel to BCE in Fig. 2. being drawn through the ellipsis from the points (Fig. 1.)

$v, w,$

$v, w, x, y, z, \beta, \gamma, \delta, \epsilon, \lambda, \pi$, will cut the ellipsis in the points 1 11, 2 10, 3 9, &c. through which the hour-lines on the horizontal dial must be drawn from the center C .]

The point C (Fig. 3.) from which the hour-lines are drawn, should not coincide with the center O of the dial-plate, but be taken at some small distance from it, toward the left-hand from XII in the meridian line, in order to enlarge the spaces between the hours near mid-day, as the angular distances between them are less than those about VI in the morning or afternoon.

From the point E in Fig. 1. draw $E I$ perpendicular to $E H$, till it meets the axis FCG (produced beyond G) in I . Then, as $H\beta E$ represents a horizontal plane, so $E I$ will represent a vertical one, facing the south; and serve for shewing how to draw the hour-lines on an erect direct south dial, as Fig. 4.

From the point I in Fig. 1. draw $IHGE$ perpendicular to EI : and make GH in Fig. 4. equal to EI in Fig. 1. With this extent, as a radius, set one foot of the compasses in G (Fig. 4.) and with the other foot describe the semicircle $H12E$, and divide it into 12 equal parts. Through these points of division, which are equidistant from 12, draw the straight lines 11 1, 10 2, 9 3, 8 4, and 7 5. Make $G6$ in Fig. 4. equal to CD in Fig. 1. and with that extent, as a radius, set one foot of the compasses in G (Fig. 4.) and with the other foot describe the semicircle $6g6$. Then divide this semicircle into 12 equal parts, and thro' these points of division draw straight lines parallel to Gg , meeting the former straight lines (drawn parallel to HE) in the points 7 5, 8 4, 9 3, &c. and draw the semi-ellipsis 6 7 8 9 10 11 12 1 2 3 4 5 6 through these points. So the longest diameter $G12$ of this semi-ellipsis shall be equal to IE in

Fig.

Fig. 1. and its shortest diameter $6G6$ equal to DCE . This done, lay the edge of a ruler to the center G (Fig. 4.) and successively to all the division-points 7, 8, 9, &c. in the semi-ellipsis, and draw straight lines from G thro' these points, as in the Figure, and they will be the true hour-lines on the erect direct south dial; to which set the hours, as in Fig. 4. and draw GC therein parallel to GC in Fig. 1. for the hypotenuse or axis of the stile that casts a shadow on the time of the day; and the dial will be finished when the stile GCK is erected upon the 12 o'clock line $GgK12$. And when the dial is truly set, the edge GC of the stile will be parallel to the axis of the world.

[N. B. Straight lines drawn parallel to HGE , thro' the semi-ellipsis 6126 , from the points (Fig. 1.) 7, 8, 9, 10, 11, in the perpendicular EI , where the lines lm , no , pq , rs , and tu , cut this

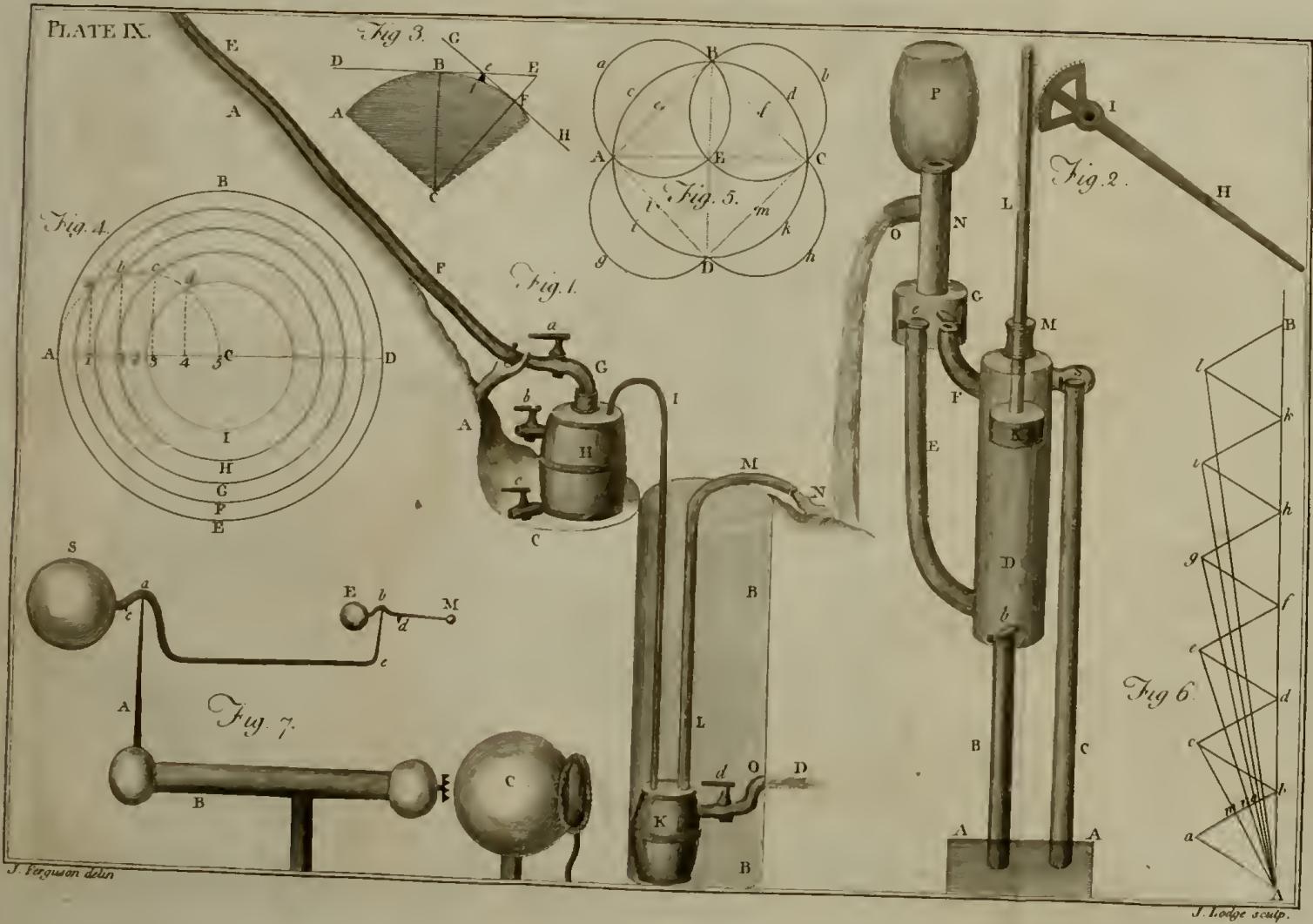
perpendicular, will cut the semi-ellipsis in Fig. 4. in those points through which the hour-lines must be drawn from the center G.]

And thus, by means of a circle divided into 24 equal parts, as in Fig. 1. a horizontal dial or erect direct south dial may be made for any latitude. The method may perhaps appear tedious, on account of the number of words in the description: but I will venture to say, that whoever puts it in practice, will find it short, easy, and pleasant.

Description of the Hungarian Machine for raising Water from Mines.

In Fig. 1. of Plate IX, *AA* is the side of a hill, close by the brink of the shaft or mine-pit *BB*, which is $10\frac{1}{4}$ feet deep below the surface of the ground *C* at the foot of the hill. In this hill is

PLATE IX.



J. Ferguson delin

is a large spring of water, 143 feet above the surface of the ground at *C* (taken in perpendicular measure) and the spring affords much more water than what the spring *D*, under ground, lets in to the mine.

A pipe *EFG* lets the water down from the spring in the hill, into a close air tight vessel *H* that stands at the foot of the hill, and contains $57\frac{1}{2}$ cubic feet, or 43 gallons in wine measure. In this pipe is a cock *a*, which being opened or shut, lets the water of the spring run into *H*, or stops it, as occasion requires: and in *H* are two cocks *b* and *c*, the uppermost of which is for letting air into *H*, and the lowermost for letting the water out of it.

A small pipe *I* goes from the vessel *H* on the surface of the ground to a vessel *K* in the bottom of the mine, and terminates in the top thereof: This vessel is air-tight, and contains $27\frac{1}{2}$ cubic feet, or $205\frac{1}{2}$ gallons in wine measure, which is forced up the

ascending pipe $L M$, and runs off to waste, at N , above ground. The lower end of this pipe goes down so far into the vessel K , as almost to touch its bottom.

From this vessel, a pipe O goes to the spring D under ground, which lets water into the mine, and would overflow it if the water was not forced up or raised from the mine, through the pipe $L M$. The pipe O lets this water into the vessel K when the cock d is turned open, and keeps back the water when the cock is shut.

The Operation is as follows:

The cock b being open, and the cocks a and c shut, and no water in the vessel K , open the cock d to let the vessel K fill with water from the spring in the mine. As this vessel fills, the water will drive the air out of it, up through the small pipe I into the vessel H , and all that air will go out of the

the vessel *H* by the open cock *b*, and then *H* will remain, as it was before, full of air in the same state of density as the common air is on the outside of *H*. When *K* is full of water, shut the cocks *b* and *d*, and open the cock *a* to let water run down from the spring in the hill, by the pipe *EF*, into the vessel *H*. As the water rises in that vessel, the air will thereby be driven out of it, down through the pipe *I*, into the vessel *K*: and as this air is compressed by the weight of the running water in the pipe *EF*, the compressed air will force all the water out of the vessel *K*, up through the pipe *LM*, from which it will run off at *N* on the surface of the ground; and then the compressed air will rush out, after the water.

When the vessel *K* in the mine is emptied of water, and the air is heard to begin to rush out, shut the cock *a* to stop the water from the spring, and open the cocks *b* and *c*: then the water that came from the spring will

run out of the vessel *H* by the cock *c*, and air will go in by the cock *b*: at the same time, open the cock *d* in the mine, to let the vessel *K* fill with water from *D* the spring in the mine; and as *H* empties above-ground, *K* will fill below it; and the air that remained in *K* will (by the rising of the water in it) be driven back into the vessel *H* through the pipe *I*.

When *H* is empty of water, and *K* full, shut the cocks *b*, *c*, and *d*, and open the cock *a*: then *H* will fill with water from the spring in the hill; and this water, as it rises in *H*, will force the air out of *H*, down the pipe *I*, upon the water in *K*; and the force of the compressed air will drive all the water out of *K*, up the pipe *LM*, from which it will run off at *N*, as before.

And thus, wherever there is a spring in a hill, near a mine, that affords more water than what flows into the mine from a spring under ground; and the perpendicular height of the

spring

spring in the hill is greater than the depth of the mine; water may thus be raised from the mine, in a most simple and easy manner, by an engine in which there are neither pumps, pistons, nor valves: and such an engine will not be liable to be out of order, nor need repairs in many years. As this engine is but very little known in Britain, I have made a working model of it, which I always shew in my course of lectures.

But as there are very few mines that have hills near them with high springs, water cannot then be raised from them in this manner; and therefore Mr. Blakey proposed another method, which was, to make *H* an air-vessel, with a pipe going from it to another vessel in which is water, kept boiling by a fire under it, and this vessel to have a cock to let out the steam occasionally that rises from the surface of the boiling water. When the cock is shut, the steam will go off from the boiler

boiler into the air-vessel *H*, and drive the air out of it, down through the pipe *I* into the vessel *K* in the mine: and the force of the air compressed by the elasticity of the steam, will raise the water from *K*, up through the pipe *LM*, till *K* be emptied of water. Then the cock in the boiler is to be turned open, to let out the steam, and the cock *d* to be opened to let the vessel *K* fill from the spring in the mine: and when it is full, both these cocks are to be shut, and the operation will go on as before.

That Blakey's scheme would do, the Hungarian machine puts beyond all doubt.—In both of them the vessels must be made very strong, because every part of each vessel, equal in surface to the bore of the ascending pipe *LM*, will sustain an outward pressure equal to the whole weight of water in that pipe.—It will not answer for such depths as the common fire-engine will, nor will it raise so much water;

water; but it may be built for less than a third part of the expence, and would answer very well where the depth is not above 100 feet.

Description of a pump, invented by M. De-la-Hire, which raises Water equally quick by the Descent as by the Ascent of the Piston in the Pump-Barrel.

In Fig. 2. of Plate IX. *AA* is the well, in which the lower ends of the pipes *B* and *C* are placed. *D* is the pump-barrel, into the lowermost end of which the top of the open pipe *B* is soldered, and in the uppermost end the hollow piece *S* is soldered, which opens into the barrel, and the top of the pipe *C* is soldered into that piece. Each of these pipes has a valve on its top, and so have the crooked pipes *E* and *F*, whose lower ends are open into the pump-barrel, and their upper ends into the box *G*.

H is

H is the pump-handle, its center of motion is at *I*; and as it is moved up and down, it moves the solid plunger *K* up and down in the barrel, by the straight rod or spear *L*, which moves air-tight in a long collar of leathers in the neck *M*; and the plunger never goes higher than *K*, nor lower than *D*; so that from *K* to *D* is the length of the stroke.

As the plunger rises from *D* to *K*, the atmosphere (pressing on the surface of the water *AA* in the well) forces the water up the pipe *B*, through the valve *b*, and fills the pump-barrel with water up to the plunger: and during this time, the valves *e* and *S* lie close and air-tight on the tops of the pipes *E* and *C*.

When the plunger is up to its greatest height at *K*, it stops there for an instant; and in that instant the valve *b* falls, and stops the pipe *B* at top. Then, as the plunger goes down, it cannot force the water between *K* and *D* back through

through the close valve *b*, but forces all *that* water up through the crooked pipe *E* through the valve *e*, which then opens upward by the force of the water; and this water, after having filled the box *G*, rises into the pipe *N*, and runs off by the spout at *O*.

During the descent of the plunger *K*, the valve *f* falls down, and covers the top of the crooked pipe *F*; and the pressure of the atmosphere on the well *AA* forces water up the pipe *C*, through the valve *S*, which then opens upward by the force of the ascending water; and this water runs from *S* into the pump-barrel, and fills all the space in it above the plunger.

When the plunger is down to its lowest descent at *D*, and stops there for an instant, in that instant the valve *S* falls down, and shuts the top of the pipe *C*: and then, as the plunger is raised, it cannot force the water above it back through the valve *S*, but drives all that water up the crooked pipe *F*, through

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through the valve *f*, which opens upward by the force of the ascending water; which water, after filling the box *G*, is forced up from thence into the pipe *N*, and runs off by the spout at *O*.

And thus; as the plunger descends, it forces the water below it up the pipe *E*; and as it ascends, it forces the water above it up the pipe *F*; the pressure of the atmosphere filling the pump-barrel below the plunger thro' the pipe *B* while the plunger ascends, and filling the barrel with water above the plunger, through the pipe *C*, as the plunger goes down.

And thus, there is as much water forced up the pipe *N* to the spout *O* by the descent of the plunger, as by its ascent; and, in each case, as much water discharged at *O* as fills that part of the pump-barrel as the plunger moves up and down in.

On the top of the pipe *O* is a close air-vessel *P*. When the water is forced up

up above the spout *O*, it compresses the air in the vessel *P*; and this air, by the force of its spring acting on the water, causes the water to run off by the spout *O* in a constant (and very nearly) equal stream.

Whatever the height of the spout *O* be above the surface of the well, the top *S* of the pipe *C* must not be 32 feet above that surface; because, if that pipe could be entirely exhausted of air, the pressure of the atmosphere in the well would not force the water up the pipe to a greater height than 32 feet. And if *S* be within 24 feet of the surface of the well, the pump will be so much the better.

As the collar of leathers within the neck *M* are apt to dry and shrink when the pump is not used, and consequently to let air get into the pump-barrel, which would stop the operation of the atmosphere in the pipe *C*, I think collars of old hats might be used instead of leathers, as they would not be liable to that inconvenience.

It matters little what the size of the pipe *N* be, through which the water is forced up to the spout: but a great deal depends on the size of the pump-barrel; and, according to the height of the spout *O* above the surface of the well, the diameter of the bore of the barrel should be as follows.

For 10 feet high, the bore should be 6.9 inches; for 15 feet, 5.6; for 20 feet, 4.9; for 25 feet, 4.4; for 30 feet, 4.0; for 35 feet, 3.7 inches; for 40 feet, 3.5; for 45 feet, 3.3; for 50 feet, 3.1; for 55 feet, 2.9; for 60 feet, 2.8; for 65 feet, 2.7; for 70 feet, 2.6; for 75 feet, 2.5; for 80 feet, 2.5 will do; for 85 feet, 2.4; for 90 feet, 2.3; for 95 feet, 2.2; and for 100 feet, the diameter of the bore should not exceed 2.1 or 2.2 inches at most.—If these proportions are attended to, a man of common strength may raise water 100 feet high by one pump as easily as he could raise it 10 feet high by another.

In this pump, the pipes *B* and *C* seem to be rather too small, which will

will cause the water rising in them to have a great deal of friction, from the quickness of its motion: and whoever makes such a pump, will find it very difficult to make the leathers in the neck *M* water-tight, so as that no water shall be forced out that way when the piston is drawn up.

The Height of the apparent Level above the true.

In Fig. 3. of Plate IX. let *ABF* be part of the Earth's spherical surface, *C* the Earth's center, *BC* or *FC* its semidiameter, *DBE* a tangent to the Earth's surface at *B*, drawn perpendicular to *BC*; and *GFH* a tangent to the Earth's surface at *F*, drawn perpendicular to *FC*. The line *DBE* is a true level at *B*; but being carried on straight toward *D* or *E*, it rises above the Earth's surface: and although it seems to be level as seen

from B , it is above the true level at F , by the whole height FE ; for, at the point F , the tangent GH is the true level.

At the distance of a geographical mile, or 6094 feet, from the point B , the line BD or BE will be 10.637 inches above the globular surface of the Earth: at two such miles from B , the same line will be four times as high (or 3 feet 6.548 inches) above the Earth's surface: at three miles distance, nine times as high (or 7 feet 11.733 inches) above the Earth's surface; and so on, always increasing in height according to the square of the distance: for, if F be twice the distance of f from B , FE must be four times as high as fe , if both their tops touch the right line BeE .

At the distance of an English mile, or 5280 feet, the apparent level is 7.90 inches higher than the true; at two miles distance, it is four times as high, or 2 feet 7.60 inches above the true;

at

at three miles distance, nine times as high; and so on, increasing in proportion to the square of the distance.

At the distance of a degree, or 60 geographical miles, which are equal to $69\frac{1}{4}$ English miles, the height of the apparent level above the true is 3191 feet and $\frac{9}{10}$ parts of an inch. And therefore, a hill, whose top was so far above the level of the sea, would be just seen at top by an eye just at the surface of the sea, and $69\frac{1}{4}$ English miles from the hill.

Of the Velocities acquired by falling Bodies, and the Spaces they fall through in given Times.

In successive equal parts of time, as 1, 1, 1, 1, &c. the spaces thro' which a body falls are as 1, 3, 5, 7, &c. and the acquired velocities are as 1, 2, 3, 4, &c. continually: so that the velocities

are as the times, and the spaces are as the squares of the times in falling.

Thus, in the first second of time (from the instant of beginning to fall) the body will fall through 16 feet; in the next second it will fall through three times 16, or 48 feet, which added to the former 16 makes 64 feet, the whole space fallen through in 2 seconds of time: in the third second of time, the body falls 5 times 16, or 80 feet; which, added to the above 64, makes 144 feet, the whole space fallen through in 3 seconds: in the fourth second it falls 7 times 16, or 112 feet; which, added to the above 144 feet, makes 256, the whole space fallen through in 4 seconds: and so on continually, increasing as the odd numbers 1, 3, 5, 7, 9, 11, in 1, 2, 3, 4, 5, 6 seconds of time.

Whatever velocity the body acquires at the end of the first second, it will acquire twice as much at the end of the next, three times as much at the end

end of the third, four times as much at the end of the fourth second, and so on continually.

In the following Table, the numbers under T denote the seconds of time, from 1 to 60, in which the body continues to fall: the numbers under S denote the spaces, in feet, through which the body falls in any second from 1 to 60: and the numbers under N denote the whole number of feet the body falls through, at the end of any number of seconds from 1 to 60. Thus, between the end of the 59th and 60th second, the body falls 119 feet; and at the end of the 60th second it has fallen through 57600 feet.

In a quarter of a second from the instant of beginning to fall, a body would fall one foot: at the end of half that second it will have fallen 4 feet: at the end of three quarters of that second it will have fallen through 9 feet: and at the end of that whole second, through 16.

The whole spaces fallen through being as the squares of the times in which the body falls, Qu. How many feet would it fall through in an hour?

In 60 seconds the space is 57600 feet, and the square of 60 is 3600. But 57600 multiplied by 3600 is 207360000, the number of feet the body would fall through in an hour, in a free or unresisting space: and this number being divided by 5280, the number of feet in an English mile, quotes 39272.7 for the number of miles.

TABLE of FALLING BODIES.

T.	S.	N.	T.	S.	N.	T.	S.	N.	T.	S.	N.
1	1	Feet 16	16	31	4096	31	61	15376	46	91	33856
2	3	64	17	33	4624	32	63	16384	47	93	35344
3	5	144	18	35	5184	33	65	17424	48	95	36864
4	7	256	19	37	5776	34	67	18496	49	97	38416
5	9	400	20	39	6400	35	69	19600	50	99	40000
6	11	576	21	41	7056	36	71	20736	51	101	41616
7	13	784	22	43	7744	37	73	21904	52	103	43264
8	15	1024	23	45	8464	38	75	23104	53	105	44944
9	17	1296	24	47	9216	39	77	24336	54	107	46656
10	19	1600	25	49	10000	40	79	25600	55	109	48400
11	21	1936	26	51	10816	41	81	26896	56	111	50176
12	23	2304	27	53	11664	42	83	28224	57	113	51984
13	25	2704	28	55	12544	43	85	29584	58	115	53824
14	27	3136	29	57	13456	44	87	30976	59	117	55696
15	29	3600	30	59	14400	45	89	32400	60	119	57600

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A TABLE shewing how much the Mercury would sink in a Barometer at given Heights above the Earth's plane Surface; and consequently, how the Heights of Hills may be found thereby.

At the height of	Merc. sinks 100 parts.	At the height of	Merc. sinks 100 parts.	At the height of	Merc. sinks 100 parts.	At the height of	Merc. sinks 100 parts.	At the height of	Merc. sinks 100 parts.
Feet.	Feet.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.	Feet.	Inches.
100	0 .11	3700	3 .83	7300	7 .05	10900	9 .87	14500	12 .37
200	0 .22	3800	3 .92	7400	7 .13	11000	9 .94	14600	12 .44
300	0 .33	3900	4 .02	7500	7 .22	11100	10 .01	14700	12 .50
400	0 .44	4000	4 .12	7600	7 .30	11200	10 .08	14800	12 .57
500	0 .54	4100	4 .21	7700	7 .38	11300	10 .16	14900	12 .63
600	0 .65	4200	4 .30	7800	7 .46	11400	10 .23	15000	12 .70
700	0 .76	4300	4 .39	7900	7 .55	11500	10 .30	15100	12 .76
800	0 .87	4400	4 .49	8000	7 .63	11600	10 .37	15200	12 .83
900	0 .98	4500	4 .58	8100	7 .71	11700	10 .44	15300	12 .89
1000	1 .09	4600	4 .67	8200	7 .79	11800	10 .52	15400	12 .96
1100	1 .19	4700	4 .77	8300	7 .87	11900	10 .59	15500	13 .02
1200	1 .30	4800	4 .86	8400	7 .95	12000	10 .66	15600	13 .09
1300	1 .40	4900	4 .95	8500	8 .03	12100	10 .73	15700	13 .15
1400	1 .50	5000	5 .04	8600	8 .11	12200	10 .80	15800	13 .21
1500	1 .61	5100	5 .13	8700	8 .19	12300	10 .87	15900	13 .28
1600	1 .72	5200	5 .22	8800	8 .27	12400	10 .94	16000	13 .34
1700	1 .82	5300	5 .31	8900	8 .35	12500	11 .01	16100	13 .40
1800	1 .93	5400	5 .40	9000	8 .43	12600	11 .08	16200	13 .47
1900	2 .03	5500	5 .49	9100	8 .51	12700	11 .15	16300	13 .53
2000	2 .14	5600	5 .58	9200	8 .58	12800	11 .22	16400	13 .59
2100	2 .24	5700	5 .67	9300	8 .66	12900	11 .29	16500	13 .65
2200	2 .34	5800	5 .76	9400	8 .74	13000	11 .36	16600	13 .71
2300	2 .44	5900	5 .85	9500	8 .82	13100	11 .43	16700	13 .78
2400	2 .54	6000	5 .94	9600	8 .89	13200	11 .50	16800	13 .84
2500	2 .65	6100	6 .02	9700	8 .97	13300	11 .56	16900	13 .90
2600	2 .75	6200	6 .11	9800	9 .05	13400	11 .63	17000	13 .96
2700	2 .85	6300	6 .20	9900	9 .12	13500	11 .70	17100	14 .02
2800	2 .95	6400	6 .28	10000	9 .20	13600	11 .77	17200	14 .08
2900	3 .05	6500	6 .37	10100	9 .27	13700	11 .84	17300	14 .15
3000	3 .15	6600	6 .45	10200	9 .34	13800	11 .90	17400	14 .21
3100	3 .25	6700	6 .54	10300	9 .42	13900	11 .97	17500	14 .27
3200	3 .34	6800	6 .63	10400	9 .50	14000	12 .04	17600	14 .33
3300	3 .44	6900	6 .71	10500	9 .57	14100	12 .11	17700	14 .39
3400	3 .54	7000	6 .80	10600	9 .64	14200	12 .17	17800	14 .45
3500	3 .63	7100	6 .88	10700	9 .72	14300	12 .24	17900	14 .51
3600	3 .73	7200	6 .97	10800	9 .79	14400	12 .30	18000	14 .57

By

By this Table, and a common barometer, the height of any hill may be found, if its height, taken in perpendicular measure, be not much above half a mile.—Thus, if the mercury be 2 inches and 95 hundred parts of an inch lower in the tube at the top of the hill, than what it was observed to be at the bottom, the perpendicular height of the hill is 2800 feet, which is 160 feet more than half a mile.

But as there are many hills much higher than 2800 feet, and the common barometer-scale is only 3 inches long, let a scale $1\frac{1}{4}$ inches long, divided into inches, and hundred parts of an inch by diagonal lines, be applied to the tube, and have a sliding index across it in the common way: and this, I apprehend, will do for the highest mountain on the earth.—For, supposing the quicksilver was observed to be 13 inches and 21 hundred parts of an inch lower in the tube when at the

the top of the hill than it was when at the foot: against 13.21 inches in the Table is 15800 feet for the height of the hill, which wants only 40 feet of being 3 miles high.

To divide the Area of a given Circle into any required Number of equal Parts, by concentric Circles.

In Fig. 4. of Plate IX. let $ABDE$ be a circle, whose area is required to be divided into 5 equal parts by concentric circles, as F, G, H, I .

Divide the semidiameter AC into 5 equal parts, as $1, 2, 3, 4, 5$; and on the middle point e as a center, with the radius eA , describe the semicircle $AabcdeC$. From the points of equal division at $1, 2, 3$, and 4 , and perpendicular to AC , raise the perpendiculars $1a, 2b, 3c, 4d$, till they meet the semicircle in the points a, b, c , and d : through which points, draw

the

the concentric circles *F*, *G*, *H*, *I*, and the thing will be done.

Supposing that five blacksmiths should agree to buy a grinding-stone among them, each paying an equal share of the price, and that each man should therefore have the use of the stone, to wear off a fifth part of it, till it came to the last man, who was to wear it out: the first man should wear the stone from *E* to *F*, the second from *F* to *G*, the third from *G* to *H*, the fourth from *H* to *I*, and the fifth from *I* to the center or axle *C*.

By this easy method, which I learnt of Mr. Hutton, teacher of the Mathematics at Newcastle, the area of any circle may be divided by concentric circles into any required number of equal parts. For, into whatever number of equal parts the radius *AC* be divided, the area of the circle will be divided into the like number of parts, all equal among themselves.

To make two equal Circles, whose Areas, taken together, shall be equal to the Area of a given Circle: or four equal Crescents, the Sum of whose Areas shall be equal to the Area of a given Square.

In Fig. 5. of Plate IX. let $cdki$ be the given circle. In this circle describe the square $efml$; and, on the middle points of any two of its sides, as at e and f as centers, describe the two circles $AaBEA$ and $BbCEB$: the areas of these two circles, taken together, shall be equal to the area of the given circle $cdki$.

Draw the diagonal AEC , which will divide the square into two triangles ABC and ADC , right angled at B and D . Now, as the sides AB and BC of the triangle ABC are equal, and so are the sides AD and DC of the triangle ADC , and the areas of circles being as the squares of their diameters,

meters, and the hypotenuse AC being squared is equal to the squares of the sides AB and BC , or AD and DC ; the larger semicircle $A c B d C$ is equal to the two lesser semicircles $A a B e A$ and $B d C f B$. Consequently, if you subtract the two common portions $A c B e A$ and $B d C f B$, the two remaining crescents $A a B c A$ and $B b C d B$ will be equal to the two triangles $A E B A$ and $B C E B$, which make one half of the square $e f m l$: and therefore the sum of the areas of all the four outward crescents is equal to the area of the whole square.

Of squaring the Circle.

Although there has not yet been any method found for doing this to mathematical exactness, yet, by means of the following numbers, it may be brought so very near the truth, as to be within a grain of sand in a square mile,

mile, supposing 100 grains of sand (placed in a straight line, and touching one another) to be equal to the length of an inch; and consequently 40144896000000 to cover a square mile.

If the diameter of a circle be given, and the length of the side of a square so nearly equal to the circle as to be true to 14 places of figures be required; say, As 1 is to the diameter of the given circle, so is 0.88622692545276 to the side of the square required, in such measures as the diameter of the circle was taken.

If the length of the side of a square be given, and the diameter of a circle equal (as nearly as above mentioned) to the square be required; say, As 1 is to the side of the given square, taken in any measure, as feet, inches, &c. so is 1.12837916709551 to the diameter of a circle (taken in the same kind of measures) whose area is equal to the area of the given square.

In practice, it is sufficient to take out the decimal parts to four places of

of figures; for, even by so small a number, we come so near the truth as to be within a ten thousandth part of the whole area of being perfectly true. And this is nearer than any one can pretend to delineate on paper.

Thus, supposing the diameter of a circle to be 12 inches, and that it is required to find the length of the side of a square (or to make a square) whose area shall be equal to the area of the circle; say, As 1 is to 12, so is .8862 to 10.6344 inches, the length of the side of the square required.

Or, supposing the side of a square to be 12 inches, and that it is required to find the diameter of a circle whose area shall be equal to the area of a square; say, As 1 is to 12, so is 1.1284 (instead of 1.128379) to 13.5408 inches, the diameter of the circle required.

Hence, as a square vessel, just one foot wide and one foot deep, would hold a cubic foot of water; a cylindrical vessel 13.54 inches wide and one foot deep would a cubic foot of water too;

too ; at least so near the truth, that no difference could be perceived.

The diameter of any circle is in proportion to its circumference, as 1 is to 3.1415926535897932384626434 ; or as 1 is to 3.1416, near enough for practice.

Any circle is equal to a parallelogram, whose length is equal to half the circumference of the circle, and breadth equal to half the diameter. Therefore multiply half the circumference by half the diameter, and the product shall be equal to the area of the circle, in square measure. The square root of this area is the side of a square equal to the circle.

To shew that an Angle may be continually diminished, and yet never be reduced to nothing: and consequently, that Matter is infinitely divisible.

In Fig. 6. of Plate IX. let AB be a straight line, produced to an infinite length beyond B , and straight throughout. On this line let there be an infinite number of equilateral triangles placed, as Aab , bcd , def , fgb , &c. whose bases Ab , bd , df , fb , &c. touch one another upon the right line AB : and let the side ab of the first triangle be of any given length, as suppose an inch, and each side of each triangle be of the same length with ab .

Then, from the point A draw the straight line Ac to the top of the second triangle bcd ; and Ac shall cut ab in the middle point at m .

From the point A draw the straight line Ae to the top of the third triangle def ; and Ae shall cut ab at n , in two

thirds of its length from a , leaving only one third remaining, from n to b .

From the point A draw the right line Ag to the top of the fourth triangle fgh ; and Ag shall cut ab at o , in three fourth parts of its length from a ; and consequently leave one fourth of it remaining, from o to b .

Here it is plain, that every line drawn from A to the top of the next triangle beyond that to which the last preceding line was drawn, will make a less angle with the line AB than the last preceding line did. But no right line drawn from the point A to the top of any triangle placed upon AB , even at an infinite distance from A , could ever coincide with the line AB , although every succeeding line will make a less angle with AB than the line last drawn before it did: and therefore the angle at A will be continually diminishing, but can never come to nothing. Consequently, the

whole line $a\,b$ will never be exhausted or quite cut off by any line drawn from A to the top of any triangle: and therefore, a part of it will still remain between a and b ; which proves that matter is infinitely divisible.

A New Experiment in Electricity, shewing the Motions of the Sun, Earth, and Moon; by Edward King, Esq; of Lincoln's Inn.

The Sun and Earth go round the common center of gravity between them in a solar year, and the Earth and Moon go round the common center of gravity between them in a lunar month.—These motions are represented by an electrical experiment, as follows:

In Fig. 7. of Plate IX. the ball S represents the Sun, E the Earth, and M the Moon, connected by bended wires $a\,c$ and $b\,d$: a is the center of gravity between

between the Sun and Earth, and b is the center of gravity between the Earth and Moon. These three balls, and their connecting wires, are hung and supported on the sharp point of a wire A , which is stuck upright in the prime conductor B of the electrical machine; the Earth and Moon hanging upon the sharp point of the wire cae , in which wire is a pointed short pin, sticking out horizontally at c ; and there is just such another pin at d , sticking out in the same manner, in the wire that connects the Earth and Moon.

When the globe C of the electrical machine is turned, the above-mentioned balls and wires are electrified: and the electrical fire, flying off horizontally from the points c and d , cause S and E to move round their common center of gravity a ; and E and M to move round their common center of gravity b . And, as E and M are light when compared with S and E , there

is much less friction on the point *b* than upon the point *a*; so that *E* and *M* will make many more revolutions about the point *b* than *S* and *E* make about the point *a*.—I had this experiment from my ingenious friend Mr. King; and have adjusted the weights of the balls so, that *E* and *M* go twelve times round *b* in the time that *S* and *E* go only once round *a*.—It makes a good amusing experiment in electricity; but is so far from proving that the motions of the planets in the Heavens are owing to a like cause, that it plainly proves they are not. For the real Sun and Planets are not connected by wires or bars of metal; and consequently there can be no such metallic points as *a* and *b* between them. And without such points, the electric fluid would never cause them to move: for, take away these points in the above-mentioned experiment, and the balls will continue at rest, let them be ever so strongly electrified.

T A B L E S

F O R

Calculating THE TRUE TIME of any
NEW or FULL MOON,
from the Creation of the World
to A. D. 7800; near enough the
Truth for any common Almanack.

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In all the British Lunar Tables hitherto published, the time depending upon the Moon's annual equation is subtracted from the mean time of New and Full Moon when the Sun's anomaly is less than six signs, and added when greater: in the elliptic equation, the time depending on the Moon's anomaly is added to the mean time of New and Full Moon when her anomaly is less than six signs, and subtracted therefrom when her anomaly is greater. In the following Tables, I have made these equations always additive, which renders the calculations much easier. For this purpose, I have put down all the radical mean times of New and Full Moon 13 hours 59 minutes sooner than in the former Tables; the greatest annual equation being 4 hours 11 minutes, and the greatest elliptic equation 9 hours 48 minutes; the sum of both these (to the nearest full minute) is 13 hours 59 minutes. The numbers under A are the degrees of the Sun's mean anomaly, and those under B the degrees of the Moon's.—I was led to this, by observing that the late eminent M. CLAIRAUT at Paris has made all the equations additive in his Lunar Tables.

TABLE I. *The mean Times of New and Full Moon in January, from A. D. 1700 to A. D. 1800, according to the Old Stile.*

Years.	New Moon.				Full Moon.					
	D.	h.	m.	A	B	D.	h.	m.	A	B
L. 1700	8	0	43	202	1	22	19	5	217	194
1701	26	22	16	221	336	12	3	54	206	143
1702	16	7	5	210	286	1	12	43	195	93
1703	5	15	53	199	236	20	10	15	214	69
L. 1704	23	13	26	218	211	8	19	4	203	18
1705	12	22	15	207	161	27	16	37	222	354
1706	2	7	3	196	111	17	1	25	211	304
1707	21	4	36	215	87	6	10	14	200	254
L. 1708	9	13	24	203	36	24	7	46	218	229
1709	28	10	57	222	12	15	16	35	207	179
1710	17	19	46	211	322	3	1	24	196	129
1711	7	4	34	200	272	21	22	56	215	105
L. 1712	25	2	7	219	247	10	7	45	204	55
1713	14	10	56	208	197	29	5	18	223	30
1714	3	19	44	197	147	18	4	6	212	340
1715	22	17	17	216	122	7	22	55	201	290
L. 1716	11	2	5	205	72	25	20	27	220	265
1717	0	10	54	194	22	15	5	16	209	215
1718	19	8	27	213	358	4	14	5	198	165
1719	8	17	15	202	308	23	11	37	217	140
L. 1720	26	14	48	220	283	11	20	26	205	90
1721	15	23	37	209	233	1	5	15	194	40
1722	5	8	25	198	183	20	2	47	213	16
1723	24	5	58	217	158	9	11	36	202	325
L. 1724	12	14	47	206	108	27	9	9	221	301
1725	1	23	35	195	58	16	17	57	210	251
1726	20	21	8	214	34	6	2	46	199	201
1727	10	5	56	203	344	25	0	18	218	177
L. 1728	28	3	29	222	319	13	9	7	207	126
1729	17	12	18	211	269	2	17	56	196	76
1730	6	21	6	200	219	21	15	28	215	52
1731	25	18	38	219	194	11	0	16	204	1
L. 1732	14	3	28	208	144	28	21	49	223	337

TABLE I. (*Mean Times of New and Full Moon in January, Old Style*) continued.

Years.	New Moon.					Full Moon.				
	D.	h.	m.	A	B	D.	h.	m.	A	B
1733	3	12	16	197	94	18	6	38	212	287
1734	22	9	49	216	69	7	15	27	201	236
1735	11	18	37	205	19	26	12	59	220	212
L. 1736	0	3	26	194	329	14	21	48	209	162
1737	19	0	59	213	305	4	6	37	198	112
1738	8	9	47	202	254	23	4	9	217	87
1739	27	7	20	220	230	12	12	58	205	37
L. 1740	15	16	9	209	180	0	21	47	194	347
1741	5	0	57	198	130	19	19	19	213	323
1742	23	22	30	217	105	9	4	8	202	273
L. 1743	13	7	18	206	55	28	1	40	221	248
1744	1	16	7	195	5	16	20	29	210	198
1745	20	13	40	214	341	5	19	18	199	148
1746	9	22	28	203	291	24	16	50	218	124
1747	28	20	1	222	266	14	1	39	207	73
L. 1748	17	4	50	211	216	2	10	28	196	23
1749	6	13	38	200	166	21	8	0	215	359
1750	25	11	11	218	141	10	16	49	203	308
1751	14	19	59	207	91	0	1	37	192	258
L. 1752	3	4	48	196	41	17	23	10	211	234
1753	22	2	21	215	16	7	7	59	200	184
1754	11	11	9	204	326	26	5	31	219	159
1755	0	19	58	193	276	15	14	20	208	109
L. 1756	18	17	31	212	252	3	23	9	197	59
1757	8	2	19	201	202	22	20	41	216	34
1758	26	23	52	220	177	12	5	30	205	344
1759	16	8	41	209	127	1	14	19	194	294
L. 1760	4	17	29	198	77	19	11	51	213	270
1761	23	15	2	217	52	8	20	40	202	219
1762	12	23	50	206	2	27	18	12	221	195
1763	2	8	39	195	312	17	3	1	210	145
L. 1764	20	6	12	214	287	5	11	50	199	95
1765	9	15	0	203	237	24	9	22	218	70
1766	28	12	33	221	213	13	18	11	206	20

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TABLE I. (Mean Times of New and Full Moon in January, Old Style) concluded.

Years.	New Moon.					Full Moon.				
	D.	h.	m.	A	B	D.	h.	m.	A	B
1767	17	21	22	210	163	3	3	0	195	330
L. 1768	6	6	10	199	113	21	0	32	214	306
1769	25	3	43	218	88	10	9	21	203	255
1770	14	12	31	207	38	29	6	53	222	231
L. 1771	3	21	20	196	348	18	15	42	211	181
1772	21	18	53	215	323	7	0	31	200	131
1773	11	3	41	204	273	25	22	3	219	106
1774	0	12	30	193	223	15	6	52	208	56
L. 1775	19	10	3	212	198	4	15	41	197	6
1776	7	18	51	201	148	22	13	13	216	341
1777	26	16	24	220	124	11	22	2	205	291
1778	16	1	12	209	74	1	6	50	194	241
L. 1779	5	10	1	198	24	20	4	23	213	217
1780	23	7	34	217	359	8	13	12	202	167
1781	12	16	22	206	309	27	10	44	221	142
1782	2	1	11	195	259	16	19	33	210	92
L. 1783	20	22	44	213	234	6	4	22	198	42
1784	9	7	32	202	184	24	1	54	217	17
1785	28	5	5	221	160	13	10	43	206	327
1786	17	13	54	210	110	2	19	32	195	277
L. 1787	6	22	42	199	59	21	17	4	214	252
1788	24	20	15	218	35	10	1	53	203	202
1789	14	5	3	207	345	28	23	25	222	178
1790	3	13	52	196	295	18	8	14	211	128
L. 1791	22	11	25	215	270	7	17	3	200	78
1792	10	20	13	204	220	25	14	35	219	53
1793	0	5	2	193	170	14	23	24	208	3
1794	19	2	35	212	145	4	8	13	197	313
1795	8	11	23	201	95	23	5	45	216	288
L. 1796	26	8	56	219	71	11	14	34	204	238
1797	15	17	44	208	21	0	23	22	193	188
1798	5	2	33	197	331	19	20	55	212	164
1799	24	0	6	216	306	9	5	44	201	114
L. 1800	12	8	54	205	256	27	3	16	220	89

TABLE II. *The mean Times of New and Full Moon in January, from A.D. 1752 to A.D. 1800, according to the New Style.*

Years.	New Moon.					Full Moon.				
	D.	h.	m.	A	B	D.	h.	m.	A	B
1753	3	13	37	186	350	18	7	59	200	184
1754	22	11	9	204	326	7	16	47	189	133
1755	11	19	58	193	276	26	14	20	208	109
L. 1756	29	17	31	212	252	14	23	9	197	59
1757	19	2	19	201	201	4	7	57	186	9
1758	8	11	8	190	151	23	5	30	205	344
L. 1759	27	8	41	209	127	12	14	19	194	294
L. 1760	15	17	29	198	77	0	23	7	183	244
1761	5	2	18	187	27	19	20	40	202	219
1762	23	23	50	206	2	9	5	28	191	169
L. 1763	13	8	39	195	312	28	3	1	210	145
L. 1764	1	17	28	184	262	16	11	50	199	95
1765	20	15	0	203	237	5	20	38	188	45
1766	9	23	49	202	187	24	18	11	206	20
1767	28	21	22	210	163	14	3	0	195	330
L. 1768	17	6	10	199	113	2	11	48	184	280
1769	6	14	59	188	63	21	9	21	203	255
1770	25	12	31	207	38	10	18	9	192	205
L. 1771	14	21	20	196	348	0	2	58	181	155
L. 1772	3	6	9	185	298	18	0	31	200	131
1773	22	3	41	204	273	7	9	19	189	80
1774	11	12	30	193	223	26	6	52	208	56
L. 1775	0	21	19	182	173	15	15	41	197	6
L. 1776	18	18	51	201	148	4	0	29	186	316
1777	8	3	40	190	198	22	22	2	205	191
1778	27	1	12	209	74	12	6	50	194	241
1779	16	10	1	198	24	1	15	39	183	191
L. 1780	4	18	50	187	334	19	13	12	202	167
1781	23	16	22	206	309	8	22	0	191	117
1782	13	1	11	195	259	27	19	33	210	92
1783	2	10	0	184	209	17	4	22	198	42
L. 1784	20	7	32	203	184	5	13	10	187	352
1785	9	16	21	192	134	24	10	43	206	327

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 TABLE II. concluded. *New Stile.*

Years.	New Moon.					Full Moon.				
	D.	h.	m.	A	B	D.	h.	m.	A	B
1786	28	13	54	210	110	13	19	32	195	277
1787	17	22	42	199	59	3	4	20	184	227
L. 1788	6	7	31	188	9	21	1	53	203	202
1789	25	5	3	207	345	10	10	41	192	152
1790	14	13	52	196	295	29	8	14	211	128
L. 1791	3	22	41	185	245	18	17	3	200	78
1792	21	20	13	204	220	7	1	51	189	28
1793	11	5	2	193	170	25	23	24	208	3
1794	0	13	51	182	120	15	8	13	197	313
L. 1795	19	11	23	201	95	4	17	1	186	263
1796	7	20	12	190	45	22	14	34	204	238
1797	26	17	44	208	21	11	23	22	193	188
1798	16	2	33	197	331	1	8	11	182	138
1799	5	11	22	186	281	20	5	44	201	114
C. 1800	24	8	54	205	256	9	14	32	190	63

 TABLE III. *Mean Lunations.*

Lun.	D.	h.	m.	A	B	Months.		Days to be subtracted.
						For	Subt.	
1	29	12	44	29	26	January	-	0
2	59	1	28	58	52	February	-	31
3	88	14	12	87	77	March	-	59
4	118	2	56	116	103	April	-	90
5	147	15	40	146	129	May	-	120
6	177	4	24	175	155	June	-	151
7	206	17	8	204	181	July	-	181
8	236	5	52	233	207	August	-	212
9	265	18	36	262	232	September	-	243
10	295	7	20	291	258	October	-	273
11	324	20	5	320	284	November	-	304
12	354	8	49	349	310	December	-	334
13	383	21	33	18	336			

In Leap years, in January and February, add a day to the time found by these Tables.

TABLE IV. *The first Equation. (A)*

A	h.	m.	A	h.	m.	A	h.	m.	A	h.	m.	A	h.	m.
1	4	7	37	1	43	73	0	13	109	0	12	145	1	45
2	4	2	38	1	39	74	0	12	110	0	14	146	1	48
3	3	58	39	1	36	75	0	11	111	0	15	147	1	52
4	3	54	40	1	32	76	0	10	112	0	17	148	1	56
5	3	50	41	1	29	77	0	9	113	0	18	149	1	59
6	3	46	42	1	26	78	0	8	114	0	20	150	2	3
7	3	42	43	1	23	79	0	7	115	0	22	151	2	7
8	3	37	44	1	19	80	0	6	116	0	23	152	2	11
9	3	33	45	1	16	81	0	5	117	0	25	153	2	15
10	3	29	46	1	13	82	0	4	118	0	27	154	2	19
11	3	24	47	1	10	83	0	3	119	0	29	155	2	23
12	3	20	48	1	7	84	0	2	120	0	31	156	2	27
13	3	16	49	1	4	85	0	2	121	0	34	157	2	31
14	3	12	50	1	1	86	0	1	122	0	36	158	2	35
15	3	7	51	0	58	87	0	1	123	0	38	159	2	39
16	3	3	52	0	56	88	0	1	124	0	40	160	2	43
17	2	59	53	0	53	89	0	0	125	0	43	161	2	48
18	2	55	54	0	51	90	0	0	126	0	45	162	2	52
19	2	51	55	0	48	91	0	0	127	0	48	163	2	56
20	2	47	56	0	45	92	0	0	128	0	51	164	3	0
21	2	43	57	0	43	93	0	0	129	0	53	165	3	5
22	2	39	58	0	40	94	0	0	130	0	55	166	3	9
23	2	35	59	0	38	95	0	1	131	0	59	167	3	13
24	2	31	60	0	36	96	0	1	132	1	2	168	3	18
25	2	27	61	0	34	97	0	1	133	1	5	169	3	22
26	2	23	62	0	32	98	0	2	134	1	8	170	3	27
27	2	19	63	0	30	99	0	2	135	1	11	171	3	31
28	2	15	64	0	28	100	0	3	136	1	14	172	3	35
29	2	12	65	0	26	101	0	4	137	1	17	173	3	40
30	2	8	66	0	24	102	0	5	138	1	20	174	3	44
31	2	4	67	0	22	103	0	6	139	1	24	175	3	49
32	2	0	68	0	20	104	0	7	140	1	27	176	3	53
33	1	57	69	0	19	105	0	8	141	1	30	177	3	58
34	1	54	70	0	17	106	0	9	142	1	34	178	4	2
35	1	50	71	0	15	107	0	10	143	1	37	179	4	7
36	1	46	72	0	14	108	0	11	144	1	41	180	4	11

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TABLE IV. (*Equation A*) concluded.

A	h.	m.												
181	4	15	217	6	45	253	8	12	289	8	7	325	6	32
182	4	20	218	6	48	254	8	13	290	8	5	326	6	29
183	4	24	219	6	52	255	8	14	291	8	3	327	6	25
184	4	29	220	6	55	256	8	15	292	8	2	328	6	22
185	4	33	221	6	58	257	8	16	293	8	0	329	6	18
186	4	38	222	7	2	258	8	17	294	7	58	330	6	14
187	4	42	223	7	5	259	8	18	295	7	56	331	6	10
188	4	47	224	7	8	260	8	19	296	7	54	332	6	7
189	4	51	225	7	11	261	8	20	297	7	52	333	6	3
190	4	55	226	7	14	262	8	20	298	7	50	334	5	59
191	5	0	227	7	17	263	8	21	299	7	48	335	5	55
192	5	4	228	7	20	264	8	21	300	7	46	336	5	51
193	5	9	229	7	23	265	8	21	301	7	44	337	5	47
194	5	13	230	7	26	266	8	22	302	7	41	338	5	43
195	5	17	231	7	29	267	8	22	303	7	39	339	5	39
196	5	22	232	7	31	268	8	22	304	7	37	340	5	35
197	5	26	233	7	34	269	8	22	305	7	34	341	5	31
198	5	30	234	7	37	270	8	22	306	7	31	342	5	27
199	5	34	235	7	39	271	8	22	307	7	29	343	5	22
200	5	39	236	7	41	272	8	21	308	7	26	344	5	19
201	5	43	237	7	44	273	8	21	309	7	23	345	5	15
202	5	47	238	7	46	274	8	20	310	7	21	346	5	10
203	5	51	239	7	48	275	8	20	311	7	18	347	5	6
204	5	55	240	7	50	276	8	19	312	7	15	348	5	2
205	5	59	241	7	53	277	8	19	313	7	12	349	4	58
206	6	3	242	7	55	278	8	18	314	7	9	350	4	54
207	6	7	243	7	57	279	8	17	315	7	6	351	4	49
208	6	11	244	7	59	280	8	16	316	7	3	352	4	45
209	6	15	245	8	0	281	8	15	317	7	0	353	4	41
210	6	19	246	8	2	282	8	14	318	6	56	354	4	37
211	6	23	247	8	4	283	8	13	319	6	53	355	4	34
212	6	26	248	8	6	284	8	12	320	6	50	356	4	28
213	6	30	249	8	7	285	8	11	321	6	46	357	4	24
214	6	34	250	8	9	286	8	10	322	6	43	358	4	20
215	6	37	251	8	10	287	8	9	323	6	39	359	4	15
216	6	41	252	8	11	288	8	8	324	6	36	360	4	11

TABLE V. *The second Equation. (B)*

B	h. m.	B	h. m.	B	h. m.	B	h. m.	B	h. m.	B	h. m.
1	9 59	37	16 2	73	19 21	109	18 51	145	15 4		
2	10 10	38	16 11	74	19 23	110	18 47	146	14 56		
3	10 21	39	16 19	75	19 25	111	18 43	147	14 48		
4	10 32	40	16 27	76	19 27	112	18 38	148	14 39		
5	10 43	41	16 35	77	19 29	113	18 34	149	14 31		
6	10 54	42	16 43	78	19 30	114	18 29	150	14 23		
7	11 5	43	16 50	79	19 32	115	18 24	151	14 14		
8	11 16	44	16 58	80	19 33	116	18 19	152	14 5		
9	11 27	45	17 5	81	19 34	117	18 14	153	13 57		
10	11 38	46	17 12	82	19 35	118	18 8	154	13 48		
11	11 49	47	17 19	83	19 35	119	18 3	155	13 39		
12	11 59	48	17 26	84	19 36	120	17 57	156	13 31		
13	12 10	49	17 33	85	19 36	121	17 51	157	13 22		
14	12 20	50	17 39	86	19 36	122	17 45	158	13 13		
15	12 31	51	17 46	87	19 36	123	17 40	159	13 4		
16	12 42	52	17 52	88	19 36	124	17 34	160	12 55		
17	12 52	53	17 58	89	19 35	125	17 28	161	12 46		
18	13 2	54	18 4	90	19 35	126	17 22	162	12 37		
19	13 13	55	18 9	91	19 34	127	17 15	163	12 28		
20	13 23	56	18 15	92	19 33	128	17 9	164	12 18		
21	13 33	57	18 20	93	19 32	129	17 2	165	12 9		
22	13 43	58	18 25	94	19 31	130	16 56	166	12 0		
23	13 53	59	18 30	95	19 30	131	16 49	167	11 51		
24	14 3	60	18 35	96	19 28	132	16 42	168	11 41		
25	14 13	61	18 40	97	19 26	133	16 35	169	11 32		
26	14 23	62	18 44	98	19 24	134	16 28	170	11 23		
27	14 32	63	18 48	99	19 22	135	16 21	171	11 14		
28	14 42	64	18 52	100	19 20	136	16 14	172	11 4		
29	14 52	65	18 56	101	19 18	137	16 8	173	10 55		
30	15 1	66	19 0	102	19 15	138	15 59	174	10 45		
31	15 10	67	19 4	103	19 12	139	15 51	175	10 36		
32	15 19	68	19 7	104	19 9	140	15 44	176	10 26		
33	15 28	69	19 10	105	19 6	141	15 36	177	10 17		
34	15 37	70	19 13	106	19 2	142	15 28	178	10 7		
35	15 45	71	19 16	107	18 59	143	15 20	179	9 58		
36	15 54	72	19 19	108	18 55	144	15 12	180	9 48		

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TABLE V. (*Equation B*) concluded.

B	h.	m.												
181	9	38	217	4	16	253	0	38	289	0	20	325	3	51
182	9	29	218	4	8	254	0	35	290	0	23	326	3	59
183	9	19	219	4	0	255	0	31	291	0	26	327	4	8
184	9	10	220	3	52	256	0	27	292	0	29	328	4	17
185	9	0	221	3	45	257	0	24	293	0	32	329	4	26
186	8	51	222	3	37	258	0	21	294	0	36	330	4	35
187	8	41	223	3	30	259	0	19	295	0	40	331	4	44
188	8	32	224	3	22	260	0	16	296	0	44	332	4	54
189	8	22	225	3	15	261	0	14	297	0	48	333	5	2
190	8	13	226	3	8	262	0	12	298	0	52	334	5	13
191	8	4	227	3	1	263	0	10	299	0	56	335	5	23
192	7	55	228	2	54	264	0	8	300	1	1	336	5	33
193	7	45	229	2	47	265	0	6	301	1	6	337	5	43
194	7	36	230	2	40	266	0	5	302	1	11	338	5	53
195	7	27	231	2	34	267	0	4	303	1	16	339	6	3
196	7	18	232	2	27	268	0	3	304	1	22	340	6	13
197	7	8	233	2	21	269	0	2	305	1	27	341	6	23
198	6	59	234	2	14	270	0	1	306	1	32	342	6	34
199	6	50	235	2	8	271	0	1	307	1	38	343	6	44
200	6	41	236	2	2	272	0	1	308	1	44	344	6	54
201	6	32	237	1	56	273	0	0	309	1	50	345	7	5
202	6	23	238	1	51	274	0	0	310	1	57	346	7	16
203	6	14	239	1	45	275	0	0	311	2	3	347	7	26
204	6	5	240	1	39	276	0	1	312	2	10	348	7	37
205	5	57	241	1	33	277	0	1	313	2	17	349	7	48
206	5	48	242	1	28	278	0	2	314	2	24	350	7	58
207	5	39	243	1	22	279	0	3	315	2	31	351	8	2
208	5	31	244	1	17	280	0	4	316	2	38	352	8	20
209	5	22	245	1	12	281	0	6	317	2	46	353	8	31
210	5	13	246	1	7	282	0	7	318	2	53	354	8	42
211	5	5	247	1	2	283	0	9	319	3	1	355	8	53
212	4	57	248	0	58	284	0	11	320	3	9	356	9	4
213	4	49	249	0	53	285	0	13	321	3	17	357	9	15
214	4	40	250	0	49	286	0	14	322	3	25	358	9	26
215	4	32	251	0	46	287	0	16	323	3	34	359	9	37
216	4	24	252	0	42	288	0	17	324	3	42	60	9	48

TABLE VI. *Supplemental to TABLE I. for finding the mean Time of New or Full Moon in January for 6000 Years before or after any given Year in the 18th Century, according to the Julian or Old Stile.*

Years.	D.	h.	m.	A	B	Years.	D.	h.	m.	A	B
100	4	8	11	3	255	3100	16	10	41	347	253
200	8	16	22	7	151	3200	20	18	52	351	148
300	13	0	33	10	46	3300	25	3	3	354	44
400	17	8	43	13	301	3400	29	11	14	357	299
500	21	16	54	17	197	3500	4	6	40	332	169
600	26	1	5	20	92	3600	8	14	51	335	64
700	0	20	32	354	322	3700	12	23	2	338	319
800	5	4	43	358	217	3800	17	7	13	342	215
900	9	12	54	1	112	3900	21	15	24	345	110
1000	13	21	5	4	8	4000	25	23	35	349	6
1100	18	5	16	8	263	4100	0	19	1	323	235
1200	22	13	26	11	159	4200	5	3	12	326	130
1300	26	21	37	14	54	4300	9	11	23	329	26
1400	1	17	4	349	284	4400	13	19	34	333	281
1500	6	1	15	352	179	4500	18	3	45	336	177
1600	10	9	26	355	74	4600	22	11	56	340	72
1700	14	17	37	359	330	4700	26	20	7	343	327
1800	19	1	48	2	225	4800	1	15	33	317	197
1900	23	9	58	5	120	4900	5	23	44	321	92
2000	27	18	9	9	16	5000	10	7	55	324	348
2100	2	13	36	343	245	5100	14	16	6	327	243
2200	6	21	47	346	141	5200	19	0	17	331	138
2300	11	5	58	350	136	5300	23	8	28	334	34
2400	15	14	9	353	291	5400	27	16	39	337	289
2500	19	22	20	356	187	5500	2	12	5	312	159
2600	24	6	31	0	82	5600	6	20	16	315	54
2700	28	14	41	3	337	5700	11	4	27	318	209
2800	3	10	8	337	207	5800	15	12	38	322	205
2900	7	18	19	341	102	5900	19	20	49	325	100
3000	12	2	30	344	358	6000	24	5	0	328	355

To calculate the true Time of New, or Full Moon.

For any proposed year, within the limits of Table I. for Old Stile, or Table II. for New Stile, write out the mean time of New or Full Moon in January, with the numbers or arguments under A and B. With these arguments find the equations in Tab. IV. and V. which being added to the mean time of New or Full Moon in January, will give the true time thereof in that month.

For the time of New or Full Moon in any month after January, add as many lunations from Table III. to the mean time in January, as the given month is after January, and also the numbers A and B for these lunations, to the numbers A and B belonging to the mean time in January. Then, with the respective sums of these numbers, if under 360, find the corre-

L 2 sponding

sponding equations in Tab. IV. and V. which added to the mean time will give the true time of the required New or Full Moon, in days, hours, and minutes, from the beginning of January. When either of the sums A or B exceeds 360, subtract 360 from it; and with the remainder enter the corresponding Table, and take out the equation.

Then, from the number of days made by the lunations added to the New or Full Moon day in January, subtract the number of days against the given month in Table III. and the remainder will be the day of the required New or Full Moon in that month. If this number of days be equal to the number you subtract them from, the required New or Full Moon falls not in the given month, but on the last day of the month preceding it: and, when that is the case, you must add a lunation to it from Table III. with the equations A and B for

B for that lunation; and then you will have the true time of New or Full Moon in the given month.

If the number of the given month after January be equal to or exceed the number of days accounted from the beginning of January on which the New or Full Moon therein falls, you must take out one lunation more from Table III. than what answers to the number of the given month after January; otherwise you would have the New or Full Moon not in the month you want, but in the month next before it. And this will always be the case in some month or other of the year wherein the New or Full Moon in January falls before the 11th day thereof.

In leap years, in the months of January and February, the Tables give the New or Full Moon a day sooner than it really falls: and therefore, in these years and months, a day must be added to the time found by these Tables. They always begin the day

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at noon, and reckon the hours onward from that time to the noon of the following day.

EXAMPLE I. *For the true Time of New Moon in June A. D. 1772, New Stile.*

	D.	h.	m.	A	B
To New Moon in Jan. 1772, TAB. II.	3	6	9	185	298
Add 6 lunations from TAB. III. -	177	4	24	175	155
The sums are - - - -	180	10	33	300	453
First equation (A) for 360, TAB. IV.		4	11		360
Second equation (B) for 93, TAB. V.		10	32		93
The whole makes - - - -	181	10	10		
TABLE III. against June, subtract	151				
Remains the true time, <i>viz.</i> June	30	10	16		

So the true time of the required New Moon is the 30th of June, at 16 minutes past X in the evening.

EXAMPLE II. *For the true Time of Full Moon in June A. D. 1772, New Stile.*

	D.	h.	m.	A	B
To Full Moon in Jan. 1772, TAB. II.	18	0	31	200	131
Add 5 lunations from TAB. III. -	147	15	40	146	129
The sums are - - - -	165	16	11	346	260
First equation (A) for 346, TAB. IV.		5	10		
Second equation (B) for 260, TAB. V.		0	16		
The whole makes - - - -	165	21	37		
TABLE III. against June, subtract	151				
Remains the true time, <i>viz.</i> June	14	21	37		

Namely, the 15th of June, at 37 minutes past IX in the morning.

To calculate the true Time of New or Full Moon in any given Year and Month after the 18th century, which begins with A. D. 1700, and ends with A. D. 1800.

Here we must go by the Old Stile, and then reduce the time to the New, by the Table further on, shewing the number of days whereby the stiles do differ. — In Tab. I. find a year in the 18th century of the same number with that in the century proposed: then from that year, in the 18th century, write out the mean time of New or Full Moon in January, with the numbers A and B belonging thereto. This done, find a year in Table VI. which, when added to the said year in the 18th century, shall make up the number of the given year. Take out the time and numbers A and B for that year in Table VI. and add them to those in January in Table I. and the sums shall be the mean time

of New or Full Moon in January for the given year, and the arguments for finding the equations in that month. Then, for any other month in the given year, work as already taught.

When the sum of days for January exceeds 31 (as in the following example) subtract a lunation (Tab. III.) from the time, and also the numbers A and B for that lunation from the former numbers, and set down the remainders for the mean time in January, and equation-arguments for that month. If either of the numbers A or B to be subtracted be greater than the number you would subtract from, add 360 to the lesser number, and then make the subtraction.

EXAMPLE

EXAMPLE III. *For the true Time of New Moon in May A. D. 1909, Old Stile.*

To A. D. 1709 add 200, and the sum will be 1909.

	D.	h.	m.	A	B
To New Moon in Jan. 1709, TAB. I.	28	10	57	222	12
Add, for 200 years, from TAB. VI.	8	16	22	7	151
The sums are	-	-	-	229	163
Subtract 1 lunation, TAB. III.	-	29	12	29	26
Rem. mean New Moon in Jan. 1909	7	14	35	200	137
Add 4 lunations from TAB. III.	118	2	56	116	103
The sums are	-	-	-	316	240
First equation (A) for 316, TAB. IV.	7	3			
Second equation (B) for 240, TAB. V.	1	39			
The whole makes	-	-	-	120	2 13
TABLE III. against May, subtract	120				
Remains the true time, <i>viz.</i> May	6	2	13		

So the true time of the required New Moon is the 6th of May, at 13 minutes past II in the afternoon.

There was no difference between the Old Stile and the New in the year of Christ 200.—In any century after that year, to find the difference between the Old and New Stile, divide the number of the given century by 4, and (without regarding the re-

mainder when there is any) add 3 to the quotient; then subtract the sum from the number of the century, and the remainder shall be the number of days which must be added to the Old Stile time, to reduce it to the New. Thus, in the year 1909, the difference will be found to be 12 days: and therefore, in the preceding example, the day of New Moon in May will be the 18th, according to the New Stile.

—At the end of these precepts and examples, I shall subjoin a Table of these differences, which I have copied from a certain Author, who has copied much more from my book of Astronomy into one of his, without doing the common justice of acknowledging, in these cases, from whom he copied.

To calculate the true Time of New or Full Moon in any given Year and Month between the Christian æra and the 18th Century, Old Stile.

Find a year in the 18th century of the same number with that in the century proposed, and take out the mean time of New or Full Moon in January, from Table I. for the said year in the 18th century, with its numbers A and B. Then, from Table VI. take out the time and numbers A and B for as many years as, when subtracted from the above-mentioned year in the 18th century, shall leave the given year remaining. Subtract the times and numbers taken from Table IV. from those taken from Table I. and set down the remainders for the mean time of New or Full Moon in January in the given year, and the arguments A and B for finding the equations in January: and then, for any other month in that year, work as above taught.

When

When subtraction of days cannot be made (as in the following example) add a lunation, and its numbers A and B, from Table III. to the first time and numbers taken out: then subtract, and set down the remainders for the mean time of New Moon in January, and the arguments for finding the equations belonging to it.

EXAMPLE IV. *For the true Time of Full Moon in April, Old Stile, in the Year of Christ 796.*

From A. D. 1796 subtract 1000, and 796 will remain.

	D.	h.	m.	A	B
To Full Moon in Jan. A. D. 1796	11	14	34	204	298
Add 1 lunation from TABLE III.	29	12	44	29	26
The sums are - - - -	41	3	18	233	264
From which, subt. for 1000 years, } TABLE VI. } <td>13</td> <td>21</td> <td>5</td> <td>4</td> <td>8</td>	13	21	5	4	8
Rem. for Full Moon, Jan. A. D. 796	27	6	13	229	256
Add 3 lunations, for April, TAB. III.	88	14	12	87	77
The sums are - - - -	115	20	25	316	333
First equation (A) for 316, TAB. IV.		7	3		
Second equation (B) for 333, TAB. V.		5	2		
The whole makes - - - -	116	8	30		
TABLE III. for April, subtract -		90			
Remains the true time, <i>viz.</i> April		26	8 30		

So that, in April A. D. 796, the true time of Full Moon was the 28th day, at 30 min. past VIII in the evening.

To calculate the true Time of New or Full Moon in any given Year and Month before the Christian æra, according to the Old Stile.

Find a year in the 18th century, which being added to the given number of years before Christ diminished by one, shall make any number of compleat centuries.

Find this number of centuries in Table VI. and subtract the time and numbers A and B belonging to it from those in the year of the 18th century; and the remainders will be the mean time of New or Full Moon for January in the given year, and the numbers A and B belonging thereto. Then, for any other month in that year, work as above.

When

When the number of days taken from Tab. VI. exceeds the number taken from Tab. I. add a lunation. And when subtraction cannot be made in the numbers A or B. (as is the case in the 3d and 4th lines (A) in the following example) add 360 to the lesser number, and then make subtraction.

EXAMPLE V. *For the true Time of New Moon in May, Old Stile, the Year before Christ 585.*

The years 584 added to 1716, make 2300, or 23 centuries.

	D.	h.	m.	A	B
To New Moon in Jan. 1716, TAB. I.	11	2	5	205	72
Add 1 lunation from TABLE III.	29	12	44	29	26
The sums are	40	14	49	234	98
Subtract, from TAB. VI. for 2300 years	11	5	58	350	36
New Moon in Jan. bef. Christ 585	29	8	51	244	62
Add 4 lunations, for April, TAB. III.	118	2	56	116	103
The sums are, for April	147	11	47	360	165
First equation (A) for 360, TAB. IV.		4	11		
Second equation (B) for 165, TAB. V.		12	9		
The whole makes	148	4	7		
TABLE III. for May, subtract	120				
Remains the true time, <i>viz.</i> May	28	4	7		

So that the true time was the 28th of May, at 7 minutes past IV in the after-

afternoon.—At that time, a total eclipse of the Sun put an end to the long war between the Medes and Lydians, by frightening both the armies with a sudden darkness, which overspread the field of battle, just as they were ready to begin the decisive engagement.

Some chronologists believe, that the world was created at the time of the autumnal equinox, in October, the year 4007 before the year of Christ's birth, and that the Moon was full upon the third day of the creation week, which must have been Wednesday.—But, by a calculation from these Tables, I find that the said Full Moon was on Tuesday, the 23d of October, at 45 minutes past VI in the morning; and Dr. Halley's Tables make it but one minute sooner. In that year, the autumnal equinox was on Wednesday, October the 24th.

These Tables are adapted to the meridian of London; but they will answer

answer for any other, by adding 4 minutes to the time found by them for every degree eastward from the meridian of London, or subtracting 4 minutes for every degree westward. They are as near the truth as can be, by having only two equations; and I have never found them to differ half an hour from Meyer's, which have thirteen.

And thus, by a very short and easy method, the time of any New or Full Moon, within the limits of 6000 years either before or after the Christian æra, may be found, sufficiently near the truth for any common purpose.—I have tried various methods for making such tables as should render the calculations of New and Full Moons from them still shorter, but have hitherto found it impossible, unless I should have contented myself with such as would sometimes vary a whole hour from the truth.

A TABLE

SELECT EXERCISES.

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A TABLE shewing the Number of Days between the Old and New Stile in different Periods of Time.

The days in the columns marked — are to be subtracted from the Old Stile time to reduce it to the New; and those in the columns marked + are to be added to the Old Stile, in order to reduce it to the New. All the years in this Table are Leap years in the Old Stile, but only those which are marked L are Leap years in the New. The difference between the two Stiles was nothing in the 200th year after the year of Christ's birth: from that time, backward or forward, it varies 3 days in every 400 years.

Years be- before Christ. New Stile.	Days Diff.	Years be- before Christ. New Stile	Days Diff.	Years af- ter Christ. New Stile.	Days Diff.
L. 6000	47	L. 2000	17	L. 2000	13
5900	46	1900	16	2100	14
5800	45	1800	15	2200	15
5700	44	1700	14	2300	16
L. 5600	44	L. 1600	14	L. 2400	16
5500	43	1500	13	2500	17
5400	42	1400	12	2600	18
5300	41	1300	11	2700	19
L. 5200	41	L. 1200	11	L. 2800	19
5100	40	1100	10	2900	20
5000	39	1000	9	3000	21
4900	38	900	8	3100	22
L. 4800	38	L. 800	8	L. 3200	22
4700	37	700	7	3300	23
4600	36	600	6	3400	24
4500	35	500	5	3500	25
L. 4400	35	L. 400	5	L. 3600	25
4300	34	300	4	3700	26
4200	33	200	3	3800	27
4100	32	Bef. 100	2	3900	28
L. 4000	32	L. Aft. 100	2	L. 4000	28
3900	31	0	1	4100	29
3800	30	200	0	4200	30
3700	29	300	1	4300	31
L. 3600	29	L. 400	1	L. 4400	31
3500	28	500	2	4500	32
3400	27	600	3	4600	33
3300	26	700	4	4700	34
L. 3200	26	L. 800	4	L. 4800	34
3100	25	900	5	4900	35
3000	24	1000	6	5000	36
2900	23	1100	7	5100	37
L. 2800	23	L. 1200	7	L. 5200	37
2700	22	1300	8	5300	38
2600	21	1400	9	5400	39
2500	20	1500	10	5500	40
L. 2400	20	L. 1600	10	L. 5600	40
2300	19	1700	11	5700	41
2200	18	1800	12	5800	42
2100	17	1900	13	5900	43

The Weight of Gold compared with the Weights of other Materials, of equal Bulk with the Gold.

An hundred pound weight of pure fine Gold is equal in bulk to

Pound weight.
 90.3 of Guinea Gold.
 69.1 of Quicksilver.
 58.1 of Lead.
 56.5 of pure Silver.
 54.1 of coinage Silver.
 49.4 of Bismuth.
 45.8 of Copper.
 44.1 of hammered Brads.
 40.3 of cast Brads.
 40.2 of forged Iron.
 39.8 of Spring Steel.
 37.2 of Block Tin.
 35.7 of Silver Ore.
 30.5 of Gold Litharge.
 25.5 of Lapis Calaminaris.
 24.2 of Loadstone.
 21.3 of Copper Ore.
 20.8 of Sapphires.
 17.9 of Diamonds.
 16.2 of Cornelian Stone.
 16.0 of Crystal Glass.
 15.5 of Lapis Lazuli.
 15.0 of Plate Glass.
 14.5 of red Coral.
 13.8 of Iceland Crystal.
 13.8 of white Marble.
 13.4 of Rock Crystal.
 13.1 of homogeneal Pyrites.
 11.9 of Sulphur.
 9.5 of black Lead.
 5.4 of Frankincense.
 5.1 of common Water.
 5.0 of Camphire.
 4.7 of Proof Spirits.
 4.4 of pure Spirits.
 3.9 of Oil of Turpentine.
 3.7 of Æther.
 1.2 of Cork.

An hundred pound weight of coinage Gold is equal in bulk to

Pound weight.
 110.8 of pure fine Gold.
 76.6 of Quicksilver.
 64.4 of Lead.
 62.6 of pure Silver.
 59.4 of coinage Silver.
 54.7 of Bismuth.
 50.8 of Copper.
 48.3 of hammered Brads.
 44.6 of cast Brads.
 44.5 of forged Iron.
 44.0 of Spring Steel.
 41.2 of Block Tin.
 39.5 of Silver Ore.
 33.8 of Gold Litharge.
 28.2 of Lapis Calaminaris.
 26.8 of Loadstone.
 23.0 of Copper Ore.
 22.5 of Sapphires.
 19.9 of Diamonds.
 18.4 of Cornelian Stone.
 17.7 of Crystal Glass.
 17.2 of Lapis Lazuli.
 16.6 of Plate Glass.
 16.1 of red Coral.
 15.3 of Iceland Crystal.
 15.3 of white Marble.
 15.0 of Rock Crystal.
 14.6 of homogeneal Pyrites.
 13.2 of Sulphur.
 10.5 of black Lead.
 6.0 of Frankincense.
 5.6 of common Water.
 5.5 of Camphire.
 5.2 of Proof Spirits.
 4.9 of pure Spirits.
 4.3 of Oil of Turpentine.
 4.1 of Æther.
 1.3 of Cork.

A TABLE shewing the Standard Weight, Value, and comparative View of English Silver Money, from King William the First, A. D. 1066, to A. D. 1765, according to the Mint Indentures.

Dates of the several Mint Indent- ures.	Standard of the Silver at each Period.				Number of Shil- lings and Pence in the Troy Pound of Standard Silver coined at each Period.	Weight of 20 Shillings in reckon- ing of Stan- dard Silver at each of these Pe- riods.	Weight of fine Silver contained in 20 Shillings in reckon- ing at each particular Periods.
	A. D.	Fine Silver. oz. pw.	Copper Alloy. oz. pw.	sh. d.		oz. pw. gr.	
1066	11 2 0 18			21 4	11 5 0	10 8 3	
1087	11 2 0 18			20 0	12 0 0	11 2 0	
1300	11 2 0 18			20 3	11 17 1	10 19 6	
1347	11 2 0 18			22 6	10 13 8	9 17 8	
1354							
1395	11 2 0 18			25 0	8 12 0	8 17 14	
1402							
1412	11 2 0 18			32 0	7 10 0	6 18 18	
1422	11 2 0 18			30 0	8 0 0	7 8 0	
1422	11 2 0 18			37 6	6 8 0	5 18 10	
1426							
1446	11 2 0 18			30 0	8 0 0	7 8 0	
1461							
1464							
1482	11 2 0 18			37 6	6 8 0	5 18 10	
1483							
1494							
1505	11 2 0 18			40 0	6 0 0	5 11 0	
1509							
1532	11 2 0 18			45 0	5 6 16	4 18 6	
1543	10 0 2 0			48 0	5 0 0	4 3 8	
1545	6 0 6 0			48 0	5 0 0	2 10 0	
1546							
1547	4 0 8 0			48 0	5 0 0	1 13 8	
1548							
1549	6 0 6 0			72 0	3 6 16	1 13 8	
1551	3 0 9 0			72 0	3 6 16	0 16 16	
1553	11 1 0 19			60 0	4 0 0	3 13 16	
1560							
1583	11 2 0 18			60 0	4 0 0	3 14 0	
1601							
1605							
1627							
1661							
1671	11 2 0 18			62 0	3 17 10	3 11 15	
1685							
1720							
1764							

SELECT EXERCISES.

The TABLE concluded.

Dates of the several Mint Indentures.	Value of the same 20 Sh. in reckoning of our present Money.	Proportion of Money at each Period to that of our present Money.	Value of the Ounce of the then Standard Silver to that of our present Money.	Value of the Ounce of fine Silver at each Period.	Kings and Queens in these Periods.
A. D.	l. s. d.	—	sh. d.	sh. d.	—
1066	2 18 1 $\frac{1}{2}$	2.9062	5 2	1 11 $\frac{1}{8}$	Will. Conq.
1087	3 2 0	3.1000	5 2	1 9 $\frac{5}{8}$	Will. Rufus.
1300	3 1 2 $\frac{3}{4}$	3.0614	5 2	1 9 $\frac{7}{8}$	Edward I.
1347	2 15 1 $\frac{3}{8}$	2.7557	5 2	2 0 $\frac{3}{8}$	Edward III.
1354					
1395	2 9 7 $\frac{1}{4}$	2.4802	5 2	2 3	Richard II.
1402					Henry IV.
1412	1 18 9	1.9375	5 2	2 10 $\frac{5}{8}$	
1422	2 1 4	2.0666	5 2	2 8 $\frac{1}{2}$	Henry VI.
1422	1 13 0 $\frac{3}{4}$	1.6531	5 2	3 4 $\frac{1}{2}$	
1426					
1446	2 1 4	2.0666	5 2	2 8 $\frac{1}{2}$	Edward IV.
1461					
1464					
1482	1 13 0 $\frac{3}{4}$	1.6531	5 2	3 4 $\frac{1}{2}$	
1483					Edward V.
1494					Henry VII.
1505	1 11 0	1.5500	5 2	3 7 $\frac{1}{4}$	
1509					Henry VIII.
1532	1 7 6 $\frac{5}{8}$	1.3776	5 2	4 0 $\frac{5}{8}$	
1543	1 3 3 $\frac{1}{4}$	1.1635	4 7 $\frac{7}{8}$	4 9 $\frac{5}{8}$	
1545	0 13 11 $\frac{5}{8}$	0.6984	2 9 $\frac{1}{2}$	8 0	
1546					
1547	0 9 3 $\frac{3}{4}$	0.4656	1 10 $\frac{3}{8}$	12 0	Edward VI.
1548					
1549	0 9 3 $\frac{3}{4}$	0.4656	2 9 $\frac{1}{2}$	12 0	
1551	0 4 7 $\frac{7}{8}$	0.2328	1 4 $\frac{1}{4}$	24 0	
1553	1 0 6 $\frac{2}{5}$	1.0286	5 1 $\frac{3}{4}$	5 5 $\frac{1}{8}$	Mary I.
1560					Elizabeth.
1583	1 0 8	1.0333	5 2	5 4 $\frac{7}{8}$	
1601					
1605					James I.
1627					Charles I.
1661	1 0 0	1.0000	5 2	5 7	Charles II.
1671					
1685					James II.
1720					George I.
1764					George III.

Prices of Goods at the above-mentioned Times.

From *A. D.* 1000 to *A. D.* 1066. A horse 1 l. 17 s. 6 d. A cow 6 s. An ox 7 s. 6 d. A swine 2 s. A sheep 1 s. 3 d. Wheat *per* quarter 1 s. 6 d.

From *A. D.* 1066 to *A. D.* 1199. A horse 12 s. 5 d. An ox 4 s. 8 d. A sow 3 s. A colt 2 s. $4\frac{1}{2}$ d. A calf 2 s. $4\frac{1}{4}$ d. A sheep 1 s. 8 d. Wheat *per* quarter 3 s. 1 d.

From *A. D.* 1199 to *A. D.* 1307. A horse 1 l. 11 s. An ox 4 s. 8 d. A sow 3 s. A cow 17 s. $0\frac{1}{2}$ d. A lamb 4 s. A heifer 2 s. $1\frac{1}{2}$ d. A goose 1 s. $0\frac{1}{2}$ d. A cock $4\frac{1}{2}$ d. A hen 3 d. Wheat *per* quarter 1 l. 3 s. $2\frac{1}{2}$ d.

From *A. D.* 1307 to 1418. A horse 18 s. 4 d. An ox 2 l. 6 s. 1 d. A cow 7 s. 2 d. A calf 4 s. 2 d. A sheep 2 s. 7 d. A goose 9 d. A cock $3\frac{3}{4}$ d. A hen $2\frac{3}{4}$ d. Wheat *per* quarter 15 s. Ale *per* gallon $7\frac{3}{4}$ d. Day labourer's wages $4\frac{1}{4}$ d.

From *A. D.* 1418 to *A. D.* 1524. A horse 2 l. 4 s. An ox 1 l. 15 s. $8\frac{1}{2}$ d. A cow 15 s. 6 d. A colt 7 s. 8 d. A sheep 5 s. A hog 5 s. A calf 4 s. 1 d. A cock 3 d. A hen 2 d. Wheat *per* quarter 11 s. 3 d. Ale *per* gallon $2\frac{3}{4}$ d. Day labourer's wages $3\frac{3}{4}$ d.

From *A. D.* 1524 to *A. D.* 1604. An ox 1 l. 16 s. 7 d. A sheep 4 s. $3\frac{3}{4}$ d. A calf 5 s. 6 d. A lamb 4 s. $4\frac{1}{4}$ d. A goose 1 s. A capon 1 s. Beef *per* stone 11 d. Coals *per* chaldron 7 s. $9\frac{1}{4}$ d. Wheat *per* quarter 15 s.

From *A. D.* 1624 to *A. D.* 1646. A pheasant 5 s. 6 d. A turkey 3 s. 9 d. A goose 2 s. A partridge 1 s. A pullet 1 s. 6 d. A pigeon 6 d.

From *A. D.* 1730 to *A. D.* 1760. A horse 10 l. An ox 8 l. A cow 7 l. 7 s. A hog 1 l. 15 s. A sheep 1 l. 6 s. A turkey 4 s. A cock 1 s. 3 d. Seamens wages *per* day 9 d. Common labourers 1 s. 8 d.

In the preceding Table, by comparing the number of shillings in the pound

pound weight of silver in the former times with the number in the pound weight at present, it will be found that the above-mentioned articles were not so cheap as is now generally believed.

Concerning Gold.

The standard for coinage gold is 11 ounces of pure gold and 1 ounce of copper.

A cubic foot of coinage or guinea gold is worth 60117 guineas 5 shillings $2\frac{1}{2}$ d. or 63123 pounds 2 shillings $2\frac{1}{2}$ d.

The weight of a cubic foot of such gold is 1107 pounds 14 ounces Avoirdupoise; which, in Troy weight, is 1346 pounds 4 ounces 10 penny-weight 5 grains.—A lump of this gold, equal in bulk to $2059\frac{47}{100}$ cubic feet, would weigh 2772820.7 Troy pounds; and if coined into guineas, would pay the national debt.

The Number of different Ways in which all the Letters of the Alphabet might be combined, or put together, from 1 Letter to 25. Or, the Number of Changes which might be rung on any Number of Bells not exceeding the Number of Letters in the Alphabet.

Thus, 2 letters may be put 2 different ways together; 3 letters, 6 different ways; 4 letters, 24 ways; 5 letters, 120 ways; 6 letters, 720 ways; and so on, as in the following Table.

1	A	1
2	B	2
3	C	6
4	D	24
5	E	120
6	F	720
7	G	5040
8	H	40320
9	I	362880
10	K	3628800
11	L	39916800
12	M	479001600
13	N	6227020800
14	O	87178291200
15	P	1307674368000
16	Q	20922789888000
17	R	355687428096000
18	S	6402373705728000
19	T	121645100408832000
20	U	2432902008176640000
21	V	51090942171709440000
22	W	1124000727777607680000
23	X	25852016738884976640000
24	Y	620448401733239439360000
25	Z	15511210043330985984000000

Now,

Now, supposing all the 25 letters could be put down in 30 seconds of time, or each combination of them made in that time (which might be done) it would require 57461442099517020244 Julian years to make all the various combinations which these letters would admit of. And consequently, if the world had already lasted 6000 years, it would require 9576907016586170 such ages to make all these combinations, without ever stopping for one single second of time.

Supposing a square Cistern to be a Mile wide and a Mile deep, or to contain a Cubic Mile of Water; and that a Cubic Yard of Water should run off from it every Minute until it was quite emptied. Qu. How much Time would all the Water take to run out of the Cistern?

Ans. 5451776000 minutes (for so many cubic yards there are in a cubic mile) or 10365 Julian years 139 days 7 hours 20 minutes.

Concern-

Concerning the Strength of Steam. From the Reverend Mr. Mitchell's Treatise on Earthquakes.

“ There are many effects produced by the vapour of water, when intensely heated, which make it probable that the force of gunpowder is not near equal to it. The effects of an exceeding small quantity of water, upon which melted metals are accidentally poured, are such, I think, as could no ways be expected from the like quantity of gunpowder. Founders, if they are not careful, often experience these effects to their cost.— An accident of this kind happened about forty years ago, at the casting of two brass cannon at Windmill-Hill, Morefields. The heat, says Cranmer, of the metal of the first gun drove so much damp into the mould of the second, which was near it, that, as soon as the metal was let into it, it blew up

with the greatest violence, tearing up the ground some feet deep, breaking down the furnace, untiring the house, killing many spectators on the spot with the steams of the melted metal, and scalding many others in a most miserable manner."

Volcanos prove that there are fires within the earth, far below its surface; and over some of these fires there may be caverns of water. When any of the water finds its way through the bottom of such a cavern, and falls down into the fire, the water will be immediately rarefied into steam; and the elasticity thereof will heave up the ground above it, and make an earthquake. The deeper the fire is, the further will the earthquake be extended.

MATHEMATICAL
T A B L E S

F O R

Dividing the LINES on SCALES
and SECTORS.

In these Tables I have only numbered the whole degrees, the intermediate lines shewing how many parts each of them is divided into: as, where there are three such lines, they denote the degree to be divided into quarters; where two, into thirds; and where one, into halves.

SELECT EXERCISES.

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Natural Chords.

Degrees.	Chord. Parts.	Degrees.	Chord. Parts.	Degrees.	Chord. Parts.	Degrees.	Chord. Parts.
1	4.36	14	143.86	14	282.66	14	420.09
	8.72		148.21		286.98		424.35
	13.09		152.56		291.30		428.61
	17.45	9	156.91	17	295.61	25	432.87
	21.81		161.26		299.93		437.13
	26.17		165.61		304.24		441.39
2	30.53		169.96		308.55		445.65
	34.90	10	174.31	18	312.86	26	449.90
	39.26		178.66		317.17		454.15
	43.62		183.01		321.48		458.40
	47.98		187.35		325.79		462.65
3	52.35	11	191.69	19	330.09	27	466.89
	56.71		196.02		334.39		471.13
	61.07		200.37		338.69		475.37
	65.43		204.71		342.99		479.61
4	69.79	12	209.05	20	347.29	28	483.84
	74.15		213.39		351.59		488.07
	78.51		217.73		355.88		492.30
	82.87		222.07		360.18		496.53
5	87.23	13	226.40	21	364.47	29	500.76
	91.59		230.74		368.76		504.98
	95.95		235.07		373.04		509.20
	100.31		239.40		377.33		513.42
6	104.67	14	243.73	22	381.61	30	517.63
	109.03		248.06		385.89		521.85
	113.38		252.39		390.18		526.06
	117.74		256.72		394.46		530.27
7	122.09	15	261.05	23	398.73	31	534.47
	126.45		265.38		403.01		538.68
	130.80		269.70		407.28		542.88
	135.16		274.02		411.55		547.08
8	139.51	16	278.34	24	415.82	32	551.27

Natural Chords.

Degrees.	Chord. Parts.	Degrees.	Chord. Parts.	Degrees.	Chord. Parts.	Degrees.	Chord. Parts.
33	555.46	$\frac{1}{4}$	688.19	$\frac{1}{4}$	817.45	$\frac{1}{4}$	942.79
	559.65		692.23		821.43		946.63
	563.84		696.37		825.41		950.47
	568.03	41	700.41	49	829.38	57	954.31
	572.21		704.55		833.35		958.14
	576.39		708.58		837.31		961.97
34	580.56		712.71		841.27		965.79
	584.74	42	716.73	50	845.23	58	969.61
	588.92		720.81		849.18		973.43
	593.09		724.87		853.13		977.24
35	597.25		728.94		857.08		981.04
	601.41	43	733.00	51	861.02	59	984.84
	605.57		737.06		864.96		987.64
	609.72		741.11		868.89		992.43
	613.88		745.16		872.82		996.22
36	618.03	44	749.21	52	876.74	60	1000.00
	622.18		753.25		880.66		1003.77
	626.32		757.29		884.57		1007.54
	630.46		761.33		888.48		1011.31
37	634.60	45	765.36	53	892.39	61	1015.07
	638.74		769.59		896.29		1018.83
	642.87		773.42		900.19		1022.58
	647.00		777.44		904.09		1026.33
38	651.13	46	781.46	54	907.98	62	1030.07
	655.26		785.47		911.85		1033.81
	659.38		789.48		915.74		1037.54
	663.50		793.49		919.62		1041.27
39	667.61	47	797.49	55	923.49	63	1044.99
	671.72		801.49		927.35		1048.71
	675.83		805.49		931.22		1052.42
	679.94		809.48		935.08		1056.13
40	684.04	48	813.47	56	938.94	64	1059.82

SELECT EXERCISES.

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Natural Chords.

Degrees.	Chord. Parts.	Degrees.	Chord. Parts.	Degrees.	Chord. Parts.	Degrees.	Chord. Parts.
64	1063.52	1/4	1164.95	1/4	1262.03	1/4	1354.39
	1067.22		1168.49		1265.41		1357.60
	1070.91		1172.03		1268.78		1360.80
65	1074.59	72	1175.57	79	1272.15	86	1363.99
	1078.27		1179.09		1275.51		1367.18
	1081.94		1182.61		1278.87		1370.36
	1085.61		1186.13		1282.22		1373.53
66	1089.27	73	1189.64	80	1285.57	87	1376.70
	1092.93		1193.14		1288.91		1379.86
	1096.58		1196.64		1292.24		1383.02
	1100.23		1200.14		1295.57		1386.17
67	1103.87	74	1203.63	81	1298.89	88	1389.31
	1107.51		1207.11		1302.20		1392.45
	1111.14		1210.58		1305.51		1395.58
	1114.76		1214.05		1308.81		1398.70
68	1118.38	75	1217.52	82	1312.11	89	1401.81
	1121.99		1220.98		1315.40		1404.92
	1125.60		1224.43		1318.69		1408.02
	1128.21		1227.88		1321.97		1411.12
69	1132.81	76	1231.32	83	1325.24	90	1414.21
	1136.40		1234.75		1328.50		
	1139.99		1238.18		1331.76		
	1143.57		1241.60		1335.01		
70	1147.15	77	1245.02	84	1338.26		
	1150.72		1248.43		1341.50		
	1154.29		1251.84		1344.73		
	1157.85		1255.24		1347.96		
71	1161.40	78	1258.64	85	1351.18		

End of the Table of Natural Chords.

N

178 SELECT EXERCISES.

Natural Sines.

Degrees.	Sine. Parts.	Degrees.	Sine. Parts.	Degrees.	Sine. Parts.	Degrees.	Sine. Parts.
1	4.36	1/4	143.49	1/4	279.83	1/4	410.72
	8.72		147.81		284.02		414.69
	13.09		152.12		288.20		418.66
1	17.45	9	156.43	17	292.37	25	422.62
	21.81		160.74		296.54		426.57
	26.17		165.05		300.71		430.51
	30.53		169.35		304.86		434.45
2	34.90	10	173.65	18	309.02	26	438.37
	39.26		177.94		313.16		442.29
	43.62		182.23		317.30		446.20
	47.98		186.52		321.44		450.10
3	52.34	11	190.81	19	325.57	27	453.99
	56.69		195.09		329.69		457.87
	61.05		199.37		333.81		461.75
	65.40		203.64		337.92		465.61
4	69.76	12	207.91	20	342.02	28	469.47
	74.11		212.18		346.12		473.32
	78.46		216.44		350.21		477.16
	82.81		220.70		354.29		480.99
5	87.16	13	224.95	21	358.37	29	484.81
	91.50		229.20		362.44		488.62
	95.85		233.45		366.50		492.42
	100.19		237.69		370.56		496.21
6	104.53	14	241.92	22	374.61	30	500.00
	108.87		246.15		378.65		503.77
	113.20		250.38		382.68		507.54
	117.54		254.60		386.71		511.29
7	121.87	15	258.82	23	390.73	31	515.04
	126.20		263.03		394.74		518.77
	130.53		267.24		398.75		522.50
	134.85		271.44		402.75		526.21
8	139.17	16	275.64	24	406.74	32	529.92

SELECT EXERCISES.

179

Natural Sines.

Degrees.	Sine. Parts.	Degrees.	Sine. Parts.	Degrees.	Sine. Parts.	Degrees.	Sine. Parts.
33	533.61	41	646.12	49	746.06	57	831.47
	537.30		649.49		748.96		833.86
	540.97		652.76		751.84		836.29
	544.64		656.06		754.71		838.67
	548.29		659.35		757.56		841.04
	551.94		662.62		760.41		843.39
34	555.57	42	665.88	50	763.23	58	845.73
	559.19		669.13		766.04		848.05
	562.80		672.37		768.84		850.35
	566.41		675.59		771.62		852.64
	569.99		678.80		774.39		854.91
	573.58		682.00		777.15		857.17
35	577.14	43	685.18	51	779.88	59	859.41
	580.70		688.35		782.61		861.63
	584.25		691.51		785.32		863.84
	587.79		694.66		788.01		866.03
	591.31		697.79		790.69		868.20
	594.82		700.91		793.35		870.36
36	598.32	44	704.01	52	796.00	60	872.50
	601.81		707.11		798.64		874.62
	605.29		710.19		801.25		876.73
	608.76		713.25		803.86		878.82
	612.22		716.30		806.44		880.89
	615.66		719.34		809.02		882.95
38	619.09	46	722.36	54	811.57	62	884.99
	622.51		725.37		814.12		887.01
	625.92		728.37		816.64		889.02
	629.32		731.35		819.15		891.01
	632.71		734.32		821.65		892.98
	636.08		737.28		824.13		894.93
40	639.44	48	740.22	56	826.59	64	896.87
	642.79		743.14		829.04		898.79

Natural Sines.

Degrees.	Sine. Parts.	Degrees.		Sine. Parts.
		14	75	
65	900.70			963.63
	902.59	75		965.93
	904.46	76		970.30
	906.31	77		974.37
	908.14	78		978.15
	909.96	79		981.63
	911.76	80		984.81
66	913.55	81		987.69
	915.31	82		990.27
	917.06	83		992.55
	918.79	84		994.52
67	920.50	85		996.19
	922.20	90		1000.00
	923.88			
68	925.54			
	927.18			
	928.81			
	930.42			
69	932.01			
	933.58			
	935.14			
70	936.67			
	938.19			
	939.69			
71	942.64			
	945.52			
72	948.32			
	951.06			
73	953.72			
	956.30			
	958.82			
74	961.26			

End of the Table of Natural Sines.

The Secants begin where the Sines end.

As the spaces which contain the degrees of the Sines are so very small when they go beyond 70, that they could not be divided into quarters without hurting the eye, even if the whole line of sines was a foot long, nor into any thing less than halves between 70 and 75, nor in less than whole degrees from 75 to 85, nor into any degrees at all between 85 and 90; it was thought needless to put any more numbers into this Table than what are sufficient to answer the purpose for which it was intended.

SELECT EXERCISES.

181

Natural Tangents.

Degrees.	Tang. Parts.	Degrees.	Tang. Parts.	Degrees.	Tang. Parts.	Degrees.	Tang. Parts.
1	4.36	14	144.99	14	291.47	14	450.47
	8.73		149.45		296.21		455.73
1	13.09	9	153.91	17	300.97	25	461.01
	17.46		158.38		305.73		466.31
2	21.82	10	162.86	18	310.51	26	471.63
	26.19		167.34		315.30		476.98
2	30.55	11	171.83	19	320.10	27	482.34
	34.92		176.33		324.92		487.73
3	39.29	12	180.83	20	329.75	28	493.15
	43.66		185.34		334.60		498.58
3	48.03	13	189.86	21	339.45	29	504.04
	52.41		194.38		344.33		509.53
4	56.78	14	198.91	22	349.22	30	515.03
	61.16		203.45		354.12		520.57
4	65.54	15	208.00	23	359.03	31	526.13
	69.93		212.56		363.97		531.71
5	74.31	16	217.12	24	368.92	32	537.32
	78.70		221.69		373.88		542.96
5	83.09	17	226.26	25	378.87	33	548.62
	87.49		230.87		383.86		554.31
6	91.89	18	235.47	26	388.88	34	560.03
	96.29		240.08		393.91		565.77
6	100.69	19	244.70	27	398.96	35	571.55
	105.10		249.33		404.03		577.35
7	109.52	20	253.97	28	409.11	36	583.18
	113.93		258.62		414.21		589.04
7	118.36	21	263.28		419.33		594.94
	122.78		267.95		424.47		600.86
8	127.22	22	272.63		429.63		606.81
	131.65		277.32		434.81		612.80
8	136.09	23	282.03		440.01		618.82
	140.54		286.74		445.23		624.87

182 SELECT EXERCISES.

Natural Tangents.

Degrees.	Tang. Parts.	Degrees.	Tang. Parts.	Degrees.	Tang. Parts.	Degrees.	Tang. Parts.
14	630.95	14	846.56	14	1120.41	14	1496.61
	637.07		854.08		1130.29		1510.84
	643.22		861.66		1140.28		1525.25
33	649.41	41	869.29	49	1150.37	57	1539.86
	655.63		876.98		1160.56		1554.67
	661.89		884.73		1170.85		1569.69
	668.18		892.53		1181.25		1584.90
34	674.51	42	900.40	50	1191.75	58	1600.33
	680.88		908.34		1202.37		1615.98
	687.28		916.33		1213.10		1631.85
	693.72		924.39		1223.94		1647.95
35	700.21	43	932.52	51	1234.90	59	1664.28
	706.73		940.71		1245.97		1680.85
	713.29		948.96		1257.17		1697.66
	719.90		957.29		1268.49		1714.73
36	726.54	44	965.69	52	1279.94	60	1732.05
	733.23		974.16		1291.52		1749.64
	739.96		982.70		1303.23		1767.49
	746.74		991.31		1315.07		1785.63
37	753.55	45	1000.00	53	1327.04	61	1804.05
	760.42		1008.76		1339.16		1822.76
	767.33		1017.61		1351.42		1841.77
	774.28		1026.53		1363.83		1861.09
38	781.29	46	1035.53	54	1376.38	62	1880.73
	788.34		1044.61		1389.09		1900.69
	795.44		1053.78		1401.95		1920.98
	802.58		1063.03		1414.97		1941.62
39	809.78	47	1072.37	55	1428.15	63	1962.61
	817.03		1081.79		1441.49		1983.96
	824.34		1091.31		1455.01		2005.69
	831.69		1100.91		1468.70		2027.80
40	839.10	48	1110.61	56	1482.56	64	2050.30

Natural Tangents.

Degrees.	Tang. Parts.	Degrees.	Tang. Parts.	Degrees.	Tang. Parts.	Degrees.	Tang. Parts.
65	2073.21	64	2506.52	64	3124.00	64	4086.66
	2096.54		2538.65		3171.59		4165.30
	2120.30		2571.50		3220.53		4246.85
	2144.51	69	2605.09	73	3270.85	77	4331.48
	2169.17		2639.45		3322.64		4419.36
	2194.30		2674.62		3375.94		4510.71
66	2219.92		2710.62		3430.84		4605.72
	2246.04	70	2747.48	74	3487.41	78	4704.63
	2272.67		2785.23		3545.73		4807.69
	2299.84		2823.91		3605.88		4915.16
	2327.56		2863.56		3667.96		5027.34
	2355.85	71	2904.21	75	3732.05	79	5144.55
67	2384.73		2945.90		3798.27		5267.15
	2414.21		2988.68		3866.71		5395.52
	2444.33		3032.60		3937.51		5530.07
	2475.09	72	3077.68	76	4010.78	80	5671.28

As the Tangents are never laid down further than to 80 degrees on common scales, it would be needless to carry them further in this Table.

The semi-tangents may be laid down on a scale by taking out the tangents of half the number of degrees in this Table.—Thus, the semi-tangent of a whole degree is the whole tangent of half a degree: the semi-tangent of 2 degrees is the whole tangent of 1 degree: the semi-tangent of 3 degrees is the whole tangent of $1\frac{1}{2}$ degree: the semi-tangent of 4 degrees is the whole tangent of 2 degrees: and so on. They are never subdivided further than to half degrees.

Natural Secants.

Degrees.	Sec. Parts.	Degrees.	Sec. Parts.	Degrees.	Sec. Parts.	Degrees.	Sec. Parts.
0	1000.00	$\frac{1}{16}$	1260.47	$\frac{3}{4}$	1459.46	$\frac{3}{4}$	1732.67
10	1015.43	38	1269.02	47	1466.28	55	1743.45
15	1035.28		1277.78		1473.19		1754.40
16	1040.30	39	1286.76		1480.19		1765.52
17	1045.69		1295.97		1487.28		1776.81
18	1051.46	40	1305.41	48	1494.48	56	1788.29
19	1057.62		1310.22		1501.77		1799.95
20	1064.18		1315.09		1509.16		1811.80
21	1071.14		1320.02		1516.65		1823.84
22	1078.53	41	1325.01	49	1524.25	57	1836.08
23	1086.36		1330.07		1531.96		1848.51
24	1094.64		1335.19		1539.79		1861.16
25	1103.38		1340.38		1547.69		1874.01
26	1112.60	42	1345.63	50	1555.72	58	1887.08
27	1122.33		1350.95		1563.87		1900.37
28	1132.57		1356.34		1572.13		1913.88
29	1143.35		1361.80		1580.51		1927.62
30	1154.70	43	1367.33	51	1589.02	59	1941.60
	1160.59		1372.93		1597.64		1955.82
31	1166.63		1378.56		1606.39		1970.29
	1172.83		1384.34		1615.26		1985.02
32	1179.18	44	1390.16	52	1624.27	60	2000.00
	1185.69		1396.06		1633.41		2015.25
33	1192.36		1402.03		1642.68		2030.77
	1199.20		1408.08		1652.09		2046.57
34	1206.22	45	1414.21	53	1661.64	61	2062.67
	1213.41		1420.42		1671.33		2079.05
35	1220.77		1426.72		1681.17		2095.74
	1228.33		1433.09		1691.16		2112.74
36	1236.07	46	1439.56	54	1701.30	62	2130.05
	1244.00		1446.10		1711.60		2147.70
37	1252.14		1452.74		1722.05		2165.68

SELECT EXERCISES.

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Natural Secants.

Deg.	Sec. Parts.	Deg. Parts.	Sec. Parts.	Deg. Parts.	Sec. Parts.	Deg. Parts.	Sec. Parts.
63	2184.01	1/4	2585.91	3/4	3193.22	1/4	4027.23
	2202.69		2613.13	72	3236.07		4283.66
	2221.74		2640.97		3280.15		4362.99
	2241.16	68	2669.47		3325.51	77	4445.41
64	2260.97		2698.64		3372.21		4531.09
	2281.17		2728.50	73	3420.30		4620.22
	2301.79		2759.09		3469.86		4713.03
	2322.82	69	2790.43		3520.94	78	4809.73
65	2344.29		2822.54		3573.61		4910.58
	2366.20		2855.45	74	3627.96		5015.85
	2388.57		2889.20		3684.05		5125.83
	2411.42	70	2923.80		3741.98	79	5248.84
66	2434.76		2959.31		3801.83		5361.23
	2458.49		2995.74	75	3863.70		5487.40
	2482.95		3033.15		3927.71		5619.76
	2507.84	71	3071.55		3993.93	80	5758.77
67	2533.29		3111.01		4062.51		
	2559.30		3151.55	76	4133.57		The End.

Natural Rhumbs.

R.	Parts.	R.	Parts.	R.	Parts.	R.	Parts.
1/4	49.09	1/4	438.19	1/4	810.48	1/4	1151.62
	98.14		485.96		855.10		1191.40
	147.12		533.42		899.22		1230.46
	196.02	3	580.56	5	942.79	7	1268.78
1	224.82		627.36		985.82		1306.34
	293.46		673.78		1028.20		1343.12
	341.92		719.80		1069.98		1379.07
	390.18	4	765.36	6	1111.14	8	1414.21

This and all the preceding Tables are fitted to one and the same Radius on the plain scale.

A TABLE shewing in what Parallel of Latitude any given Number of Geographical Miles make a Degree of Longitude, and their Projection on a plain Scale.

Miles.	Latitude.	Nat. Chord. Parts.	Miles.	Latitude.	Nat. Chord. Parts.
60	00 00	1000.00	30	60 0	1000.00
59	10 28 $\frac{1}{2}$	182.86	29	61 6	1016.58
58	14 50	258.17	28	62 11	1032.82
57	18 11 $\frac{2}{3}$	316.22	27	63 15 $\frac{1}{3}$	1048.79
56	21 2 $\frac{1}{3}$	365.43	26	64 19 $\frac{1}{4}$	1064.60
55	23 33 $\frac{1}{2}$	408.28	25	65 22 $\frac{1}{2}$	1080.11
54	25 50 $\frac{1}{2}$	447.49	24	66 25 $\frac{1}{3}$	1095.45
53	27 57	483.00	23	67 27 $\frac{1}{2}$	1110.53
52	29 55 $\frac{2}{3}$	516.42	22	68 29 $\frac{1}{2}$	1125.49
51	31 47 $\frac{1}{4}$	547.71	21	69 30 $\frac{2}{3}$	1140.15
50	33 33 $\frac{1}{2}$	577.37	20	70 31 $\frac{2}{3}$	1154.79
49	35 15	605.57	19	71 32 $\frac{1}{3}$	1168.99
48	36 52	632.40	18	72 32 $\frac{1}{2}$	1183.21
47	38 26	658.28	17	73 32 $\frac{1}{3}$	1197.20
46	39 56 $\frac{2}{3}$	683.12	16	74 32	1211.05
45	41 24 $\frac{1}{2}$	707.08	15	75 31	1224.66
44	42 50	730.30	14	76 30 $\frac{1}{3}$	1238.27
43	44 13 $\frac{1}{2}$	752.84	13	77 29 $\frac{1}{3}$	1251.70
42	45 34 $\frac{1}{3}$	774.58	12	78 27 $\frac{2}{3}$	1264.87
41	46 53 $\frac{3}{4}$	795.82	11	79 26	1277.98
40	48 11 $\frac{1}{2}$	816.52	10	80 24 $\frac{2}{3}$	1291.07
39	49 27 $\frac{1}{2}$	836.65	9	81 22 $\frac{1}{3}$	1303.82
38	50 42 $\frac{1}{4}$	856.36	8	82 20 $\frac{1}{3}$	1316.58
37	51 55 $\frac{1}{3}$	875.52	7	83 18	1329.15
36	53 8	894.48	6	84 15	1341.50
35	54 19	912.90	5	85 13	1353.97
34	55 29	930.97	4	86 10 $\frac{2}{3}$	1366.26
33	56 38	948.69	3	87 8	1378.40
32	57 46	966.06	2	88 5 $\frac{1}{2}$	1390.47
31	58 53 $\frac{1}{2}$	983.19	1	89 2 $\frac{2}{3}$	1402.36
30	60 0	1000.00	0	90 0	1414.21

Latitudes, for the Dialing Scale.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
1	24.7	31	647.5	61	931.1
2	49.3	32	662.2	62	936.0
3	73.9	33	676.4	63	940.8
4	98.4	34	690.2	64	945.4
5	122.8	35	703.6	65	949.6
6	147.0	36	716.6	66	953.9
7	171.1	37	729.2	67	957.8
8	149.9	38	741.4	68	961.5
9	218.6	39	753.2	69	965.1
10	241.9	40	764.7	70	968.5
11	265.0	41	775.8	71	971.6
12	287.9	42	786.5	72	974.5
13	310.4	43	796.8	73	977.4
14	332.5	44	806.7	74	980.1
15	354.3	45	816.5	75	982.5
16	375.8	46	825.9	76	984.8
17	396.9	47	834.8	77	986.9
18	417.6	48	843.6	78	988.8
19	437.8	49	851.9	79	990.6
20	457.7	50	860.0	80	992.4
21	477.3	51	867.8	85	998.2
22	496.1	52	875.3	90	1000.0
23	514.6	53	882.5	No Degrees can be put into this scale between 80 and 85, nor be between 85 and 90.	
24	532.8	54	889.5		
25	550.5	55	896.2		
26	567.8	56	902.6		
27	584.6	57	908.8		
28	601.0	58	914.7		
29	616.9	59	920.3		
30	632.5	60	925.8		

Hours adapted to the preceding Table of Latitudes, for Dialing.

Hours and Minutes.		Parts.	Hours and Minutes.		Parts.
XII	0	0.0	III	0	707.1
	5	30.3		5	722.5
	10	59.1		10	737.9
	15	87.0		15	753.4
	20	113.8		20	769.0
	25	139.6		25	784.6
	30	164.5		30	800.2
	35	188.8		35	816.0
	40	212.0		40	831.9
	45	234.6		45	847.8
	50	256.7		50	863.8
	55	278.0		55	880.1
I	0	298.8	III	0	896.5
	5	319.2		5	913.1
	10	339.0		10	930.0
	15	358.4		15	947.1
	20	377.3		20	964.5
	25	395.9		25	982.0
	30	414.2		30	1000.0
	35	432.2		35	1018.3
	40	449.7		40	1036.9
	45	467.1		45	1055.8
	50	484.2		50	1075.2
	55	501.1		55	1095.0
II	0	517.7	V	0	1115.4
	5	534.1		5	1136.2
	10	550.4		10	1157.5
	15	566.4		15	1179.6
	20	582.4		20	1202.2
	25	598.3		25	1225.4
	30	614.0		30	1249.7
	35	629.6		35	1274.6
	40	645.2		40	1300.4
	45	660.8		45	1327.2
	50	676.3		50	1355.1
	55	691.7		55	1383.9
III	0	707.1	VI	0	1414.2

Inclination of Meridians.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
1	24.3	31	530.8	61	909.8
2	47.7	32	543.8	62	923.3
3	70.4	33	556.8	63	936.9
4	92.4	34	569.7	64	950.6
5	113.8	35	582.4	65	964.5
6	134.5	36	595.1	66	978.5
7	154.6	37	607.7	67	992.8
8	174.2	38	620.3	68	1007.3
9	193.4	39	632.8	69	1022.0
10	212.0	40	645.2	70	1036.9
11	230.1	41	657.7	71	1052.0
12	247.9	42	670.0	72	1067.4
13	265.3	43	682.4	73	1083.1
14	282.2	44	694.7	74	1099.1
15	298.8	45	707.1	75	1115.4
16	315.1	46	719.5	76	1132.0
17	331.1	47	731.8	77	1148.9
18	346.8	48	744.2	78	1166.3
19	362.2	49	756.5	79	1184.1
20	377.3	50	769.0	80	1202.2
21	392.2	51	781.4	81	1220.8
22	406.9	52	793.9	82	1240.0
23	421.4	53	806.5	83	1259.6
24	435.7	54	819.1	84	1279.7
25	449.7	55	831.8	85	1300.4
26	463.6	56	844.5	86	1321.8
27	477.3	57	857.4	87	1343.8
28	490.9	58	870.4	88	1366.5
29	504.4	59	883.4	89	1389.9
30	517.7	60	896.5	90	1414.2

Adapted to the preceding Line of Hours, and Line of Latitudes.

The preceding Tables are for gradu-
ating the Lines of Natural Chords,
Sines, Tangents, and Secants, &c. on
common plain Scales and Sectors.—
The following Tables are for laying
down Gunter's logarithmic Lines of
Numbers, Sines, Tangents, Versed
Sines, Meridional Parts, &c. on his
Scales.

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Numbers. The Logarithm of 1 = 0.

N.	Parts.	N.	Parts.	N.	Parts.
1.01	4.32	1.31	117.27	1.61	206.83
1.02	8.60	1.32	120.57	1.62	209.52
1.03	12.84	1.33	123.85	1.63	212.19
1.04	17.03	1.34	127.10	1.64	214.84
1.05	21.19	1.35	130.33	1.65	217.48
1.06	25.31	1.36	133.54	1.66	220.11
1.07	29.38	1.37	136.72	1.67	222.72
1.08	33.42	1.38	139.88	1.68	225.31
1.09	37.43	1.39	143.01	1.69	227.89
1.10	41.39	1.40	146.13	1.70	230.45
1.11	45.32	1.41	149.22	1.71	233.00
1.12	49.22	1.42	152.29	1.72	235.53
1.13	53.08	1.43	155.37	1.73	238.05
1.14	56.90	1.44	158.36	1.74	240.55
1.15	60.70	1.45	161.37	1.75	243.04
1.16	64.46	1.46	164.35	1.76	245.51
1.17	68.19	1.47	167.32	1.77	247.97
1.18	71.88	1.48	170.26	1.78	250.42
1.19	75.55	1.49	173.19	1.79	252.85
1.20	79.18	1.50	176.09	1.80	255.27
1.21	82.79	1.51	178.98	1.81	257.68
1.22	86.36	1.52	181.84	1.82	260.07
1.23	89.91	1.53	184.69	1.83	262.45
1.24	93.42	1.54	187.52	1.84	264.82
1.25	96.91	1.55	190.33	1.85	267.17
1.26	100.37	1.56	193.12	1.86	269.51
1.27	103.80	1.57	195.90	1.87	271.84
1.28	107.21	1.58	198.66	1.88	274.16
1.29	110.59	1.59	201.34	1.89	276.46
1.30	113.94	1.60	204.12	1.90	278.75

Gunter's Line of Numbers.

N.	Parts.	N.	Parts.	N.	Parts.
1.91	281.03	2.42	383.82	3.05	484.30
1.92	283.30	2.44	387.39	3.10	491.36
1.93	285.56	2.46	390.94	3.15	498.31
1.94	287.80	2.48	394.45	3.20	505.15
1.95	290.03	2.50	397.94	3.25	511.88
1.96	292.26	2.52	401.40	3.30	518.51
1.97	294.47	2.54	404.83	3.35	525.04
1.98	296.66	2.56	408.24	3.40	531.48
1.99	298.85	2.58	411.62	3.45	537.82
2.00	301.03	2.60	414.97	3.50	544.07
2.02	305.35	2.62	418.30	3.55	550.23
2.04	309.63	2.64	421.60	3.60	556.30
2.05	313.87	2.66	424.88	3.65	562.29
2.08	318.06	2.68	428.13	3.70	568.20
2.10	322.22	2.70	431.36	3.75	574.03
2.12	326.34	2.72	434.57	3.80	579.78
2.14	330.41	2.74	437.75	3.85	585.46
2.16	334.45	2.76	440.91	3.90	591.06
2.18	338.46	2.78	444.04	3.95	596.60
2.20	342.42	2.80	447.16	4.00	602.06
2.22	346.35	2.82	450.25	4.05	607.45
2.24	350.25	2.84	453.32	4.10	612.78
2.26	354.11	2.86	456.37	4.15	618.05
2.28	357.93	2.88	459.39	4.20	623.25
2.30	361.73	2.90	462.40	4.25	628.40
2.32	365.49	2.92	465.38	4.30	633.47
2.34	369.22	2.94	468.35	4.35	638.49
2.36	372.91	2.96	471.29	4.40	643.49
2.38	376.58	2.98	474.22	4.45	648.36
2.40	380.21	3.00	477.12	4.50	653.21

Gunter's Line of Numbers.

N.	Parts.	N.	Parts.	N.	Parts.
4.55	658.01	6.05	781.75	7.55	877.95
4.60	662.76	6.10	785.33	7.60	880.81
4.65	667.45	6.15	788.75	7.65	883.66
4.70	672.10	6.20	792.39	7.70	886.49
4.75	676.69	6.25	795.88	7.75	889.30
4.80	681.24	6.30	799.34	7.80	892.09
4.85	685.74	6.35	802.77	7.85	894.87
4.90	690.20	6.40	806.13	7.90	897.63
4.95	694.61	6.45	809.56	7.95	900.37
5.00	698.97	6.50	812.91	8.00	903.09
5.05	703.29	6.55	816.24	8.05	905.71
5.10	707.57	6.60	819.54	8.10	908.48
5.15	711.81	6.65	822.82	8.15	911.16
5.20	716.00	6.70	826.07	8.20	913.81
5.25	720.16	6.75	829.30	8.25	916.45
5.30	724.28	6.80	832.51	8.30	919.08
5.35	728.35	6.85	835.69	8.35	921.69
5.40	732.39	6.90	838.84	8.40	924.28
5.45	736.40	6.95	841.98	8.45	926.86
5.50	740.36	7.00	845.10	8.50	929.42
5.55	744.29	7.05	848.19	8.55	931.97
5.60	748.19	7.10	851.26	8.60	934.50
5.65	752.05	7.15	854.30	8.65	937.02
5.70	755.87	7.20	857.33	8.70	939.52
5.75	759.67	7.25	860.34	8.75	942.81
5.80	763.43	7.30	863.32	8.80	944.48
5.85	767.16	7.35	866.29	8.85	946.94
5.90	770.85	7.40	869.23	8.90	949.39
5.95	774.52	7.45	872.16	8.95	951.82
6.00	778.15	7.50	875.06	9.00	954.24

Gunter's Line of Numbers.

N.	Parts.	N.	Parts.	N.	Parts.
9.05	956.65	9.40	973.13	9.75	989.00
9.10	959.04	9.45	975.43	9.80	991.23
9.15	961.42	9.50	977.72	9.85	993.44
9.20	963.79	9.55	980.00	9.90	995.63
9.25	966.14	9.60	982.27	9.95	997.82
9.30	968.48	9.65	984.53	10.00	1000.00
9.35	970.81	9.70	986.77	The End.	

All the number of parts into which the double Line of Numbers (from 1 to 10) on a scale two feet long can well be divided, are inserted in this Table.

The Line of Numbers is marked with the numeral figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.—The space between 1 and 2 may be divided into 100 parts; from 2 to 3, into 50; but all the others into no more than 20 each, without hurting the eye to look at such small divisions.

But, in common practice, the spaces from the first 1 to the second 1 are divided only into 10 parts each; and after that, from 1 to 2, into 50; from 2 to 5, into 20 each; and from 5 to 10, each space is divided into no more than 10 parts; leaving the intermediate subdivisions to be estimated by the eye, which may be easily done.

In the following Tables, D. stands for Degrees, and M. for Minutes of a Degree.

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Gunter's Line of Sines.

D. M.	Parts.	D. M.	Parts.	D. M.	Parts.
0 20	2235.25	3 15	1246.47	6 10	968.91
25	2138.34	20	1235.49	15	963.10
30	2059.16	25	1224.78	20	957.38
35	1992.21	30	1214.32	25	951.72
40	1934.22	35	1204.12	30	946.14
45	1883.07	40	1194.15	35	940.63
50	1837.32	45	1184.40	40	935.19
55	1795.93	50	1174.87	45	929.82
I 0	1758.14	55	1165.54	50	924.52
5	1723.39	4 0	1156.42	55	919.28
10	1691.21	5	1147.48	7 0	914.11
15	1661.25	10	1138.72	5	908.99
20	1633.22	15	1130.13	10	903.94
25	1606.90	20	1121.71	15	898.84
30	1582.08	25	1115.10	20	894.01
35	1558.61	30	1105.36	25	889.13
40	1536.34	35	1097.40	30	884.30
45	1515.15	40	1089.60	35	879.53
50	1494.96	45	1081.92	40	874.81
55	1475.66	50	1074.39	45	870.15
2 0	1457.18	55	1066.98	50	865.53
5	1439.46	5 0	1059.70	55	860.96
10	1422.43	5	1052.64	8 0	856.44
15	1406.05	10	1045.50	5	851.97
20	1390.27	15	1038.57	10	846.67
25	1375.03	20	1031.75	15	843.17
30	1360.32	25	1025.04	20	838.84
35	1346.09	30	1018.43	25	834.55
40	1332.31	35	1011.92	30	830.30
45	1318.96	40	1005.50	35	826.09
50	1306.00	45	999.18	40	821.93
55	1293.42	50	992.96	45	817.80
3 0	1281.20	55	986.82	50	813.72
5	1269.31	6 0	980.77	55	809.67
10	1257.74	5	974.78	9 0	805.67

Gunter's Line of Sines.

D.	M.	Parts.	D.	M.	Parts.	D.	M.	Parts.
9	5	801.70	14	10	611.29	20	15	460.78
10		797.77		20	606.31		30	455.67
15		793.87		30	601.40		45	450.64
20		790.01		40	596.54	21	0	445.67
25		786.18		50	591.75		15	440.77
30		782.39	15	0	587.00		30	435.92
35		778.63		10	582.32		45	431.14
40		774.91		20	577.68	22	0	426.42
45		771.22		30	573.10		15	421.76
50		767.56		40	568.57		30	417.16
55		763.93		50	564.09		45	412.61
10	0	760.33	16	0	559.66	23	0	408.12
10		753.23		10	555.28		15	403.68
20		746.24		20	550.95		30	399.30
30		739.37		30	546.66		45	394.97
40		732.61		40	542.42	24	0	390.69
50		725.95		50	538.22		15	386.46
11	0	719.40	17	0	534.06		30	382.27
10		712.95		10	529.95		45	378.14
20		706.60		20	525.88	25	0	374.05
30		700.34		30	521.86		15	370.01
40		694.18		40	517.87		30	366.02
50		688.11		50	513.93		45	362.06
12	0	682.12	18	0	510.02	26	0	358.16
10		676.22		10	506.15		15	354.29
20		670.40		20	502.32		30	350.47
30		664.66		30	498.52		45	346.69
40		659.00		40	494.77	27	0	342.95
50		653.42		50	491.04		15	339.25
13	0	647.91	19	0	487.36		30	335.59
10		642.48		10	483.71		45	331.97
20		637.11		20	480.09	28	0	328.39
30		631.81		30	476.50		15	324.85
40		626.59		40	472.95		30	321.34
50		621.42		50	469.43		45	317.87
14	0	616.32	20	0	465.95	29	0	314.43

Gunter's Line of Sines.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
30	311.02	39	208.24	56	89.31
	307.66		205.85		86.64
	304.33		203.48		84.01
	301.03		201.13		81.43
	297.76		198.80		78.89
	294.53		196.49		76.41
31	291.33	40	194.20	58	73.97
	288.16		191.93		71.58
	285.02		187.46		69.23
	281.92		183.06		66.93
	278.84		178.73		64.68
	275.79		174.49		62.47
32	272.77	43	170.32	61	58.18
	269.78		166.22		54.07
	266.82		162.19		50.01
	263.89		158.23		46.34
	260.99		154.34		42.72
	258.11		150.51		39.27
33	255.26	45	146.76	67	35.97
	252.44		143.07		32.83
	249.64		139.44		29.85
	246.87		135.87		27.01
	244.13		132.37		24.33
	241.41		128.93		21.79
34	238.71	48	125.54	71	19.40
	236.05		122.22		17.16
	233.40		118.95		15.06
	230.78		115.75		13.10
	228.18		112.59		11.28
	225.61		109.50		9.60
35	223.06	51	106.46	78	8.05
	220.54		103.47		6.65
	218.03		100.53		5.66
	215.55		97.65		0.00
	213.09		94.82		
	210.66		92.04		

The End.

Each Degree from the beginning to the 10th may be divided into 12 parts; from 10 to 20 into 6; from 20 to 40 into 4; from 40 to 60 into 2; after that to 80 into whole Degrees; and then, between 80 and 90, only the 85th Degree can be put in.

Gunter's Line of Versed Sines.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
0	0	60	124.93	93	324.37	111	493.74
10	3.31	61	129.36	94	328.38	112	499.28
12	4.77	62	133.87	95	332.43	113	504.88
14	6.49	63	138.47	96	336.41	114	510.52
16	8.16	64	143.16	97	340.63	115	516.22
18	10.76	65	147.94	98	344.79	116	521.97
20	13.30	66	152.81	99	348.98	117	527.78
22	16.10	67	157.79	100	353.20	118	533.65
24	19.19	68	162.85	101	357.47	119	539.57
26	22.55	69	168.01	102	361.77	120	545.54
28	26.19	70	173.27	103	366.11	121	551.58
30	30.11	71	178.63	104	370.49	122	557.67
32	34.31	72	184.08	105	374.91	123	563.83
34	38.81	73	189.64	106	379.37	124	570.04
36	43.58	74	195.30	107	383.86	125	576.32
38	48.66	75	201.06	108	388.41	126	582.66
40	54.03	76	206.93	109	392.98	127	589.06
41	56.82	77	212.91	110	397.60	128	595.53
42	59.70	78	218.99	111	402.26	129	602.06
43	62.64	79	225.19	112	406.95	130	608.66
44	65.67	80	231.49	113	411.70	131	615.32
45	68.77	81	237.90	114	416.48	132	622.06
46	71.95	82	244.44	115	421.31	133	628.85
47	75.20	83	251.09	116	426.19	134	635.73
48	78.54	84	257.85	117	431.10	135	642.67
49	81.95	85	264.73	118	436.07	136	649.69
50	85.45	86	271.74	119	441.07	137	656.78
51	89.02	87	278.87	120	446.13	138	663.95
52	92.68	88	286.13	121	451.22	139	671.19
53	96.41	89	293.51	122	456.37	140	678.51
54	100.23	90	301.03	123	461.56	141	685.91
55	104.14	91	304.83	124	466.80	142	693.38
56	108.13	92	308.67	125	472.09	143	700.95
57	112.20		312.55	126	477.43	144	708.58
58	116.36		316.45	127	482.82	145	716.32
59	120.61		320.40	128	488.26	146	724.13

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Gunter's Line of Versed Sines.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
129	732.03	$\frac{1}{2}$	901.27	$\frac{1}{2}$	1106.2	$\frac{1}{2}$	1382.2
	740.02		906.29		1112.7		1391.4
130	748.10	139	911.34	148	1119.3	157	1400.6
	752.16		916.42		1125.9		1410.0
131	756.27		921.55	149	1132.6	158	1419.5
	760.12		926.69		1139.4		1429.1
132	764.55	140	931.89	150	1146.2	159	1438.8
	768.52		937.12		1153.0		1448.6
133	772.91		942.38	151	1160.0	160	1458.5
	776.57		947.67		1167.0		1468.5
134	781.37	141	953.00	152	1174.0	161	1478.7
	785.45		958.37		1181.1		1489.0
135	789.93		963.78	153	1188.3	162	1499.4
	793.97		969.21		1195.5		1510.0
136	798.60	142	974.71	154	1202.8	163	1520.6
	803.33		980.23		1210.1		1531.4
137	807.36		985.80	155	1217.5	164	1542.4
	811.18		991.39		1225.0		1553.5
138	816.24	143	997.04	156	1232.6	165	1564.7
	820.72		1002.4		1240.3		1576.1
139	825.22		1008.5	157	1248.0	166	1587.7
	829.45		1014.2		1255.8		1599.4
140	834.32	144	1020.0	158	1263.6	167	1611.3
	838.91		1025.9		1271.5		1623.3
141	843.52		1031.7	159	1279.5	168	1635.6
	848.17		1037.9		1287.6		1648.0
142	852.84	145	1043.7	160	1295.8	169	1660.6
	857.54		1049.7		1304.1		1673.4
143	862.28		1055.8	161	1312.4	170	1686.3
	867.04		1061.9		1320.8		1699.6
144	871.84	146	1068.1	162	1329.2	171	1712.9
	876.67		1074.6		1337.9		1726.5
145	881.53		1080.6	163	1346.6	172	1740.3
	886.40		1086.9		1355.3		1754.3
146	891.34	147	1093.3	164	1364.2	173	1768.6
	896.29		1099.7		1373.2		1783.1

Gunter's Line of Versed Sines.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
$\frac{1}{2}$	1797.9	$\frac{1}{2}$	1859.6	$\frac{1}{2}$	1926.2	$\frac{1}{2}$	1998.4
	1812.9		1875.8		1943.7		2017.4
166	1828.2	167	1892.3	168	1961.5		
	1843.8		1909.1		1979.7		The End.

Gunter's Line of Tangents.

D. M.	Parts.	D. M.	Parts.	D. M.	Parts.
0 30	2059.14	2 45	1318.46	5 0	1058.05
35	1992.19	50	1305.47	5	1050.83
40	1934.19	55	1292.86	10	1043.73
45	1883.04	3 0	1280.60	15	1036.75
50	1837.27	5	1268.68	20	1029.87
55	1795.87	10	1257.08	25	1023.09
1 0	1758.08	15	1245.77	30	1016.42
5	1723.31	20	1234.75	35	1009.85
10	1691.12	25	1224.00	40	1003.38
15	1661.14	30	1213.51	45	996.99
20	1633.11	35	1203.27	50	990.70
25	1606.77	40	1193.26	55	984.50
30	1581.93	45	1183.47	6 0	978.38
35	1558.43	50	1173.90	5	972.34
40	1536.15	55	1164.53	10	966.39
45	1514.95	4 0	1155.36	15	960.52
50	1494.73	5	1146.37	20	954.72
55	1475.41	10	1137.57	25	948.99
2 0	1456.92	15	1128.94	30	943.34
5	1439.17	20	1120.47	35	937.76
10	1422.12	25	1112.16	40	932.25
15	1405.72	30	1104.02	45	926.80
20	1389.91	35	1096.01	50	921.42
25	1374.65	40	1088.15	55	916.11
30	1359.91	45	1080.43	7 0	910.86
35	1345.65	50	1072.84	5	905.66
40	1331.84	55	1065.38	10	900.53

SELECT EXERCISES.

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Gunter's Line of Tangents.

D.	M.	Parts.	D.	M.	Parts.	D.	M.	Parts.	D.	Parts.
7	15	895.46	30	732.20	30	528.40	34	356.54		
	20	890.44	40	725.04	40	523.78	24	351.42		
	25	885.48	50	718.14	50	519.20		346.34		
	30	880.57	11	0 711.35	17	0 514.66		341.30		
	35	875.72	10	705.32	10	510.16		336.29		
	40	870.91	20	698.05	20	505.70	25	331.33		
	45	866.16	30	691.54	30	501.28		326.40		
	50	861.46	40	685.11	40	496.89		321.50		
	55	856.80	50	678.78	50	492.54		316.64		
8	0	852.20	12	0 672.53	18	0 488.22	26	311.82		
	5	847.64	10	666.35	10	483.94		307.03		
	10	843.12	20	660.26	20	479.69		302.26		
	15	838.65	30	654.24	30	475.48		297.53		
	20	834.23	40	648.30	40	471.30	27	292.83		
	25	829.84	50	642.43	50	467.15		288.16		
	30	824.64	13	0 636.64	19	0 463.03		283.52		
	35	821.20	10	630.91	10	458.94		278.91		
	40	816.94	20	625.24	20	454.88	28	274.33		
	45	812.72	30	619.65	30	450.85		269.77		
	50	808.54	40	614.11	40	446.85		265.24		
	55	804.39	50	608.64	50	442.88		260.73		
9	0	800.29	14	0 603.23	20	0 438.93	29	256.25		
	5	796.22	10	597.88		433.07		251.79		
	10	792.18	20	592.58		427.26		247.36		
	15	788.18	30	587.34		421.51		242.95		
	20	784.22	40	582.16	21	415.82	30	238.56		
	25	780.29	50	577.03		410.19		234.20		
	30	776.39	15	0 571.95		404.60		229.85		
	35	772.53	10	566.92		399.07		225.53		
	40	768.70	20	561.94	22	393.59	31	221.23		
	45	764.90	30	557.01		388.16		216.94		
	50	761.13	40	552.13		382.78		212.68		
	55	757.39	50	547.29		377.44		208.44		
10	0	753.68	16	0 542.50	23	372.15	32	204.21		
	10	746.35	10	537.76		366.90		200.00		
	20	739.14	20	533.06		361.70		195.81		

Q

Gunter's Line of Tangents.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
33	191.64	36	138.74	40	87.76	43	37.95
	187.48		134.76		83.90		34.14
	183.34		130.79		80.04		30.34
	179.22		126.83		76.19		26.55
34	175.11	37	122.89		72.34		22.75
	171.01		118.95		68.50		18.96
	166.93		115.02		64.67		15.16
	162.87		111.00		60.84		11.37
35	158.81	38	107.19	41	57.01		7.58
	154.77		103.29		53.19		3.79
	150.75		99.39		49.38		0.00
	146.73		95.51		45.56		The End.
	142.73	39	91.63		41.75		

On Gunter's Scale, the Tangents begin at 0 degrees 35 minutes, at the left hand, and are thence numbered on to 45 degrees at the other end of the Scale; and thence backward, at the same divisions, to 89 degrees 25 minutes. For, as the Tangents above and below 45 degrees are equally distant from the Radius, every grand division may be doubly numbered. So that 40 and 50, 60 and 30, 70 and 20, 80 and 10, 85 and 5, are placed at the same grand divisions of the Tangent line.

N. B. The first three numbers (0 d. 20 m. 25 m. and 30 m.) in the preceding Table of Sines, are superfluous, because they cannot be brought into the Scale.

Gunter's Lines of Rhumbs.

Deg.	M.	Rhumbs.	Sines.	Rhumbs.	Tangents.
0	0	0	Infinite.	0	Infinite.
2	48 $\frac{3}{4}$	$\frac{1}{4}$	1309.21	8	1308.68
5	37 $\frac{1}{2}$		1008.70		1006.60
8	26 $\frac{1}{4}$		833.48		828.75
11	15	1	709.76	7	701.34
14	3 $\frac{3}{4}$		614.43		601.21
16	52 $\frac{1}{2}$		537.18		518.06
19	41 $\frac{1}{4}$		472.51		446.35
22	30	2	417.16	6	382.78
25	18 $\frac{3}{4}$		369.01		325.17
28	7 $\frac{1}{2}$		326.61		272.04
30	56 $\frac{1}{4}$		288.95		222.30
33	45	3	255.26	5	175.11
36	33 $\frac{3}{4}$		224.97		129.80
39	22 $\frac{1}{2}$		197.64		85.83
42	11 $\frac{1}{4}$		172.92		42.71
45	0	4	150.51	4	0.00
47	48 $\frac{3}{4}$		130.21		
50	37 $\frac{1}{2}$		111.81		
53	26 $\frac{1}{4}$		95.17		
56	15	5	80.15		
59	3 $\frac{3}{4}$		66.65		
61	5 $\frac{1}{2}$		54.57		
64	41 $\frac{1}{4}$		43.84		
67	30	6	34.38		
70	18 $\frac{3}{4}$		26.16		
73	7 $\frac{1}{2}$		19.11		
75	56 $\frac{1}{4}$		13.21		
78	45	7	8.43		
81	33 $\frac{3}{4}$		4.73		
84	22 $\frac{1}{2}$		2.10		
87	11 $\frac{1}{4}$		0.52		
90	0	8	0.00		

The Tangents and Co-tangents being equally distant from Radius, they are both found on the same point of the line: and therefore each grand division is doubly figured; as 1, 7, are put to signify both one point and seven points of the Compafs; and so on.

Gunter's Line of Meridional Parts.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
0	0.0	16	194.5	28	350.2	$\frac{2}{3}$	503.8
	6.0		200.8		354.7	39	508.9
1	12.0	17	207.1		359.3		514.1
	18.0		213.3	29	363.9		519.3
2	24.0	18	219.6		368.2	40	524.5
	30.0		225.8		373.1		529.7
3	36.0	19	232.3	30	377.7		535.0
	42.0		238.6		382.3	41	540.3
4	48.0	20	245.0		387.9		545.6
	54.1	$\frac{1}{3}$	249.3	31	391.6		550.9
5	60.1	$\frac{2}{3}$	253.5		396.2	42	556.3
	66.1	21	257.8		400.9		561.7
6	72.1		262.1	32	405.7		567.1
	78.2		266.4		410.4	43	572.6
7	84.2	22	270.7		415.1		578.1
	90.3		275.0	33	419.9		583.6
8	96.3		279.3		424.7	44	589.1
	102.4	23	283.7		429.5		594.7
9	108.4		288.1	34	434.3		600.3
	114.5		292.4		439.1	45	605.9
10	120.6	24	296.8		443.9		611.6
	126.7		301.1	35	448.8		617.3
11	132.8		305.5		453.7	46	623.1
	138.9	25	309.9		458.7		628.9
12	145.0		314.4	36	463.5		634.7
	151.2		318.8		468.5	47	640.5
13	157.4	26	323.3		473.5		646.4
	163.5		327.7	37	478.5		652.3
14	169.7		332.2		483.5	48	658.3
	175.9	27	336.7		488.5		664.3
15	182.1		341.1	38	493.6		670.3
	188.3		345.7		498.7	49	676.4

Gunter's Line of Meridional Parts.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
43	682.5	60	905.5	68	1126.1	76	1442.0
	688.7	4	911.5		1134.2		1454.5
50	694.9		917.5		1142.3		1467.2
	701.1		923.7		1150.5		1480.2
	707.4	61	929.8	69	1158.9	77	1493.4
51	713.7		936.0		1167.3		1506.9
	720.1		942.3		1175.8		1520.6
	726.5		948.6		1184.4		1534.6
52	733.0	62	954.9	70	1193.1	78	1548.9
	739.5		961.4		1202.0		1563.5
	746.1		967.9		1210.9		1579.3
53	752.7	63	974.4		1219.9		1593.5
	759.4		980.9	71	1229.1	79	1609.0
	766.1		987.6		1238.4		1625.0
54	772.9		994.3		1247.8		1641.3
	779.7		1001.1		1257.3		1657.9
	786.6	64	1007.9	72	1266.9	80	1675.0
55	793.5		1014.7		1276.7		1692.5
	800.5		1021.6		1286.6		1710.4
	807.6		1028.7		1296.6		1728.8
56	814.7	65	1035.7	73	1306.8	81	1747.8
	821.9		1042.8		1317.2		1767.2
	829.2		1050.1		1327.7		1787.2
57	836.5		1057.3		1338.3		1807.8
	849.9	66	1065.7	74	1349.1	82	1829.0
	851.3		1072.1		1360.1		1850.9
58	858.8		1079.6		1371.2		1873.6
	866.4		1087.1		1382.5		1896.9
	874.1	67	1094.8	75	1394.0	83	1921.1
59	881.8		1102.5		1405.7		1946.2
	889.6		1110.2		1417.6		1972.2
	897.5		1118.1		1429.7		1999.2

*Of the Construction of the plain Scale, Sector,
and Gunter's Scale.*

As it often happens that these three most useful instruments are badly divided, either through the fault of patterns erroneous in their first construction, or worn out by much use, or by the ignorance or neglect of the divider, it was thought that rules and Tables for graduating them accurately, and examining those which have been already made, might be very acceptable both to the workman and young mathematician desirous of projecting them, or to any person who is obliged to use them.

The foundation of these, and indeed of most other scales, is a line of equal parts, so subdivided as that one of the least of these parts shall scarce take up a visible space, as into thousand parts of an inch, or even more *minute*; so that

that an error of one unit in the last place, in taking off any distance, may not affect the graduations of your intended instrument. Furnished with such a scale, and a beam compass with a regulating screw at one end, you may lay down any number, of four, or even five places of figures, with great exactness.

The best way of dividing a right line into any possible number of parts, and that which is practised by our best dividers of mathematical instruments, is by help of diagonal lines; both because the smallest diagonal subdivisions are as perceptible on such a scale as the largest, and that this is the most simple method of subdividing, and may be extended further than any other way yet known.

In order to have an exact diagonal scale, it is absolutely necessary that the parallel lines, through which the diagonal lines are drawn, be equidistant among themselves; that the diagonals

be so likewise; and that, in the two outermost parallels in which the diagonals terminate, the ends of each diagonal must be just even with the opposite ends of those next to it on each side; that is, in a line perpendicular to the parallels.

In one inch, 25 diagonal lines may be very easily placed, and consequently 50 in two inches: and if these diagonals run through 20 equidistant parallel lines, they will divide every two inches of length of the diagonal scale into 1000 equal parts, which will be quite sufficient for graduating the lines of Rhumbs, Chords, Sines, Tangents, and Secants, on common plain scales, where the radius (or 60 degrees of the Chords) takes up only two inches in length of the whole line: and to that line of Chords, the Rhumbs, Sines, Tangents, and Secants, are adapted.

There is generally another line of Chords of 3 inches radius on these scales,

scales, at the right-hand end; and just above that line is a line marked *M. Lon.* for shewing how many geographical miles are contained in a degree of longitude in any given parallel of latitude. To lay down these two lines, there must be a diagonal scale containing 1000 parts in 3 inches; and these parts or divisions must be continued on to 1420, which will be sufficient for the purpose, as the whole length of each of these two lines takes up only 1414.21 parts.

But, for laying down the above-mentioned lines of Rhumbs, Sines, Chords, Tangents, and Secants; where the radius 2 inches contains 1000 equal parts, and the length of the Tangent line (in which there are only 80 degrees laid down) is $11\frac{2}{5}$ inches, the diagonal parts or equal divisions must be carried to 5760; of which, 5671.28 will be equal to the Tangent of 80 degrees, and 5758.77 to the Secant of the same number of degrees; as shewn by

by the Tables of Natural Tangents and Secants.

For Gunter's lines of Numbers, Rhumbs, Sines, Versed Sines, Tangents, and Meridional Parts, every one of which takes up $22\frac{6}{7}$ inches of his two foot scale, a diagonal scale must be first made, in which $22\frac{6}{7}$ inches shall contain 2000 equal parts: and, for this purpose, there will be no need for more than 10 equidistant parallel lines along the scale, and one inch of its length will not include quite 8 equidistant diagonal lines.

For Sectors, of whatever length the radius of the Chords (or of 60 degrees) be, which is generally very near the whole length of the Sector when shut, a diagonal scale must be prepared, in which *that* length shall contain 1000 equal parts.—The distance between these long parallels should not be less than a tenth part of an inch; otherwise the eye cannot well estimate the decimal parts expressed in the Tables, which

which are hundredth parts of the spaces between the long parallel lines, in all except the Dialing Tables of Latitudes, Hours, and Inclination of Meridians, and in the Table of Gunter's Meridional Parts; in which, the spaces between the parallels are supposed to contain only ten parts each.

Being thus provided with proper diagonal scales, patterns for graduating all the lines on Scales and Sectors may be made in the following manner.

Having drawn the lines on your intended brass pattern, analogous to those which must contain the graduations on scales, fix your diagonal scale close by the upper edge of the pattern to be graduated: and then applying one side of a square along the upper edge of the diagonal scale, and the other side directly across both that scale and the pattern, cut the first division of each line on the pattern directly even with the very beginning of the diagonals; and

and each division after *that*, against the same number of parts, found among the diagonals, as answer to the number of parts in the Tables belonging to each degree in the respective lines: and, where the lines are long enough to admit of the quarters of degrees, these may be put in, according to the numbers in the Tables.—But, in the common plain scales, where the length of the radius of the Chord line is only two inches, the spaces taken up by the degrees are too small to allow of subdivisions, except in the Tangents and Secants after the 50th degree.

Thus, in the Tables of Natural Chords, Sines, Tangents, and Secants, against the first degree is 17.45 parts both for the Chords and Sines, and 17.46 for the first degree of the Tangents. Therefore, finding those parts among the diagonals, and laying the cross edge of the square to them, cut the first degree of these lines on the pattern,

pattern, close by the said edge of the square: and proceed on in the same manner with all the rest of the degrees in the several lines, cutting them right against the same number of parts found among the diagonals which the Tables shew to belong to them respectively. Thus, the 20th degree in the line of Chords must be cut even with 347.29 parts among the diagonals; the like degree of the Sines even with 342.02 parts; and the same degree of the Tangents even with 363.97 parts in the diagonal scale.

Where the Sines end on the plain scale, the Secants begin; namely, even with 1000 parts among the diagonals. But the Secant degrees are so small at first, that there can be no putting in any of them less than the 10th, which answers to 1015.43 parts among the diagonals; and the next that can be put in is the 15th, which answers to 1035.28 parts: after which, all the degrees as far as 80 may be put into

into the line of Secants, cutting them even with the same numbers of parts found among the diagonals as stand against them in the Table of Natural Secants.

On Sectors, there are two Tangent lines on each leg. The first of these goes only to 45 degrees, the length of the leg admitting of no more: and the others begin at 45 degrees, at a fourth part of the length of the leg from the center of the joint to that part near the end of the leg where the 45th degree of the former line stands, and goes on generally to 76 degrees. The first of these, which goes from 0 degrees to 45, is called the *lower Tangents*; and the last, which goes from 45 to 76, is called the *upper Tangents*.

The lower Tangent degrees are laid down by the diagonal scale according to the numbers found against them in the Table of Natural Tangents; 1000 parts of the diagonal scale taking up just as much length as the whole 45 degrees

degrees of these Tangents do. But, as the radius of the upper Tangents is only a fourth part of the length of that of the lower ones, in order to lay them down on the Sector-pattern, all the numbers in the Table of Natural Tangents above 45 degrees must be divided by 4, and their quotients sought for, among the diagonals, for laying down the respective degrees of these Tangents. To save the operator this trouble, I have taken it, and made the following Table, which consists of these quotients, and consequently contains the number of parts (to be found among the diagonals) for dividing the line of upper Tangents, the 76th degree of which is even with 1002.69 parts of the diagonal scale.

Supplemental Table, for laying down the Line of upper Tangents on Patterns for dividing the Lines on Sectors.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
45	250.00	53	328.77	61	441.87	69	626.33
	252.19		331.76		446.41		634.46
	254.40		334.79		451.01		642.87
	256.63		337.85		455.69		651.27
46	258.88	54	340.36	62	460.44	70	659.86
	261.15		344.09		465.27		668.65
	263.44		347.27		470.18		677.65
	265.51		350.49		475.17		686.87
47	268.09	55	353.74	63	480.24	71	696.31
	270.45		357.04		485.40		705.98
	272.83		360.37		490.65		715.89
	275.23		363.75		495.99		726.05
48	277.73	56	367.17	64	501.42	72	736.47
	280.10		370.64		506.95		747.17
	282.57		374.15		512.57		758.15
	285.07		377.71		518.30		769.42
49	287.59	57	381.31	65	524.13	73	781.00
	290.14		384.96		530.07		792.70
	292.71		388.67		536.13		805.13
	295.31		392.42		542.29		817.71
50	297.94	58	396.22	66	548.57	74	830.66
	300.59		400.08		554.98		843.98
	303.27		403.99		561.51		857.71
	305.98		407.96		568.17		871.85
51	308.72	59	411.99	67	574.96	75	886.43
	311.49		416.07		581.89		901.47
	314.29		420.21		588.96		916.99
	317.12		424.41		596.18		933.01
52	319.98	60	428.68	68	603.55	76	949.57
	322.88		433.01		611.08		966.68
	325.81		437.41		618.77		984.38

The diagonal scale remaining where it did, for dividing the line of lower Tangents, [See the Remark further on] cut the 45th degree by the square against 250 parts found among the diagonals, having put the cross side of the square to it: so this second line of Tangents shall begin just at a fourth part of the length of the former line from the center of the joint; and then proceed with the rest of the degrees from 45 to 76, cutting them in the pattern even with the same number of parts found among the diagonals as belong to them in the preceding Table.

The Secants on the Sector begin also at a fourth part of the length of the leg from the center of the joint; and therefore, to lay down the degrees of the Secants on the Sector pattern, all the numbers in the Table of Natural Secants from 0 to $75\frac{1}{2}$ must be divided by 4, and their quotients taken among the parts on the diagonal scale. The following Table, for this purpose, is the quotients of the numbers in the Table of Natural Secants divided by 4.

The Secant of $75\frac{1}{4}$ degrees would reach to 1015.55, which is 15.55 parts more than the diagonal scale contains.

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Supplemental Table, for dividing the Line of Secants on Sector Patterns.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
0	250.00		315.12		364.86		433.17
10	253.86	38	317.25	47	366.57	55	435.86
15	258.82		319.44		368.30		438.60
16	260.07	39	321.69		370.05		441.38
17	261.42		323.99		371.82		444.20
18	262.86	40	326.25	48	373.62	56	447.07
19	264.40		327.55		375.44		449.99
20	266.04		328.77		377.29		452.95
21	267.78		330.00		379.16		455.96
22	269.63	41	331.25	49	381.06	57	459.02
23	271.59		332.52		382.99		462.13
24	273.66		333.80		384.95		465.29
25	275.84		335.09		386.92		468.50
26	278.15	42	336.41	50	388.93	58	471.77
27	280.58		337.74		390.79		475.09
28	283.14		339.08		393.03		478.47
29	285.84		340.45		395.13		481.90
30	288.67	43	341.83	51	397.23	59	485.40
	290.15		343.23		399.41		488.95
31	291.66		344.64		401.60		492.57
	293.21		346.08		403.81		496.25
32	294.79	44	347.54	52	406.07	60	500.00
	296.42		349.01		408.35		503.81
33	298.09		350.51		410.67		507.69
	299.80		352.02		413.02		511.64
34	301.55	45	353.55	53	415.41	61	515.67
	303.35		355.10		417.83		519.76
35	305.19		356.68		420.29		523.93
	307.08		358.25		422.79		528.18
36	309.02	46	359.89	54	425.32	62	532.51
	311.00		361.52		427.90		536.92
37	313.03		363.18		430.51		541.42

The Supplemental Table concluded.

Deg.	Parts.	Deg.	Parts.	Deg.	Parts.	Deg.	Parts.
63	546.00	60	614.02	70	705.63	73	831.33
	550.67		620.74		713.86		843.05
	555.42		626.96		722.30		855.07
	560.29		633.32		730.93		867.46
	565.24		639.82		739.83		880.23
	570.29		646.48		748.93		893.40
64	575.45	67	553.28	71	758.29	74	906.99
	580.70		660.24		767.89		921.01
	586.07		667.35		777.75		935.49
	591.55		674.66		787.89		950.46
	597.14		682.12		798.30		965.92
	602.85		689.77		809.02		981.93
65	608.69	69	697.61	72	820.01	75	998.48

On Sectors, the line of Lines (which is divided into equal parts) and the lines of Chords, Sines, Tangents, and Secants, are laid down on both the legs. They are all drawn from the center of the joint, and ought to be strictly at equal angular distances from each other, at the other ends of the legs. So that, whether the Sector be open or shut, the same opening of the compasses that reaches cross-wise from 10 on the line of Lines on one

leg to 10 on the other (at the ends furthest from the joint) should reach from 60 to 60 degrees of the Chords, from 90 to 90 of the Sines, from 45 to 45 of the lower Tangents, from end to end of the upper Tangents, and likewise from end to end of the lines of Secants. I generally find all these very well laid down, except the lines of upper Tangents and Secants ; which, for want of this precaution, are troublesome to use on most Sectors.

I apprehend that Sectors would be much more convenient than they now are, if their lines of Chords contained all the degrees of 0 to 90. For then, in laying down an angle of any number of degrees less than 90, one opening of the compasses would do ; whereas, as they now are, it requires two operations to lay down an angle of any number of degrees above 60. And besides, if the line of Chords contained all the 90 degrees, the lower Tangents, instead of ending at 45 degrees,

grees, would go on to 55, by carrying them out a very small space beyond the end of the Chords: and also, the line prolonged on which the Sines are laid down (they going no further than 60 degrees of the Chords) would receive thereon all the Secants as far as 45 degrees; so that, all these Tangents and Secants might be taken off without a second opening of the Sector, as is customary in common ones; which would be a very great convenience to those who use it. And then, beginning the line of upper Tangents at 55 degrees, and of upper Secants at 55, with a fourth part of the Tabular numbers from the center of the joint, both Tangents and Secants might be carried on to 80 degrees.—For these purposes, the diagonal scale must be so divided, as that 1414.2 of its equal parts shall be equal in length to the whole line of Chords; and then, 1440 of these parts would extend but a very little further. And

the line of Lines (which is a line of equal parts) must be so divided, as that ten of its grand divisions, to which the numeral figures are set, shall be precisely equal in length to 60 degrees of the line of Chords.—In common Sectors, 6 inches long when shut, each grand division of the line of Lines is subdivided into 20 equal parts, every one of which is supposed to be subdivided into 5; by which means, the 10 grand divisions of that line are supposed to contain 1000 equal parts, *viz.* the tabular number answering to the Radius, or 60 degrees of the Chords.

The annexed small Table is for laying down the line of Polygons on Sectors where the line of Chords goes on to 90 degrees. Thus the figure (or number) 4 must stand even with

Nº	Parts.
4	1414.21
5	1175.57
6	1000.00
7	867.89
8	765.36
9	684.04
10	618.03
11	569.55
12	517.63

1414.21 parts of the diagonal scale; the figure 5 against 1175.57 parts; the figure 6 against 1000.00 parts; and so on

on to 12, as in the Table.—But, by these numbers, the line of Polygons could be laid down only from 6 to 12 on common Sectors, where the line of Chords goes no further than 60 degrees.

But, for those who chuse to make Sectors in the common way, the here-annexed Table shews the numbers in the diagonal scale by which the line of Polygons is to be laid down. Thus, the figure 4 must answer to 1000.00 parts of the diagonal scale, the figure 5 to 813.47 parts, the figure 6 to 707.11 parts, the figure 7 to 613.69, and so on, to the last division 12.

Nº	Parts.
4	1000.00
5	813.47
6	707.11
7	613.69
8	541.19
9	483.69
10	437.01
11	402.73
12	366.02

R E M A R K.

As all those lines which are properly called *Sectoral Lines* *, terminate in an arch whose center is the center of the joint, the diagonal scale ought to be so placed, as that the long parallel lines upon it may be strictly parallel to each sectoral line on the pattern to be divided from the scale: and also, that when one side of the square is laid close to the upper edge of the diagonal scale, and the other side of the square (that lies across the scale and pattern) to the center of the joint, *that* side of the square may then be at the beginning of the diagonal divisions on the scale. Then all the divisions cut by that side of the square will be true, and each division at right angles to its own respective line. Without this precaution, the innermost sectoral lines

* The lines which are drawn from the center of the joint almost to the other ends of the legs.

would

would not be divided to their whole proper lengths: and so they would not all have the same radius, and consequently the measures taken off from them by the compasses would not agree together.

But, when the Sector-pattern is truly divided according to this method, it may be applied close to the side of the Sector to be divided from it; because, as the lines on the intended Sector will be parallel to the like lines on the pattern, one side of a square may be applied to the upper edge of the pattern, and the other side will lie across both the pattern and Sector at right angles: and then, by applying that side to each division of the pattern, and cutting each such division close by it on the Sector, all the divisions on the Sector lines will be true, although they be not cut at right angles to those lines to which they belong respectively.

As the dialing lines of Latitudes, Hours, and Inclination of Meridians, have

have no dependance on the radius of any of the above-mentioned lines, and are indiscriminately put upon Sectors and plain Scales, they may be made of any convenient length where there is sufficient room. But, as they depend upon one another, they must be all laid down from one scale of equal parts. The lines of Hours and Inclination of Meridians are of equal length, which ought to be six inches at least; and the length of the line of Latitudes is equal to that of four hours and an half, in the line of Hours.

To lay down these lines, you must have a diagonal scale of such a length, as that $14^{\frac{1}{4}}.2$ of its equal parts shall contain as great a length as the line of Hours is intended to be of. And then, the same number of parts, which stand in the Tables against the degrees of Latitudes, Inclination of Meridians, Hours and parts of an Hour, must be found among the diagonals; and the respective divisions in the lines cut by the

the side of a square applied to these parts in the diagonal scale, after it has been fixed close to the edge of that on which these lines are to be divided.— Thus, 10 degrees in the line of Latitudes (reckoned from the beginning thereof, which must be even with the beginning of the diagonal divisions) must stand even with 241.9 parts among the diagonals: 10 degrees of the line of Inclination of Meridians must be even with 212 parts of the diagonals, the hour of I against 298.8 parts; and so on, as in the Tables.

Gunter's Lines, of Rhumbs, Numbers, Sines, Versed Sines, Tangents, and Meridional Parts (on the scale that goes by his name) are all laid down by one diagonal scale of equal parts; and 2000 of these parts must include a length equal to the whole length of the line of Numbers, which consists of 18 grand divisions of different lengths, marked with the numeral figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2,

1, 2, 3, 4, 5, 6, 7, 8, 9, 10; the first grand division being the space between the first 1 and the first 2, and the last between the second 9 and the 10.— On this scale, the grand divisions from the first 1 to the second 1 are generally subdivided into 10 parts each, altho' they might bear four times that number from 1 to 3; 20 divisions each from the first figure 3 to the figure 7, and after that, only 10 divisions each, to the second figure 1, which is at the middle of the line. The grand divisions in the other half of the line are of the same lengths with those in the former half; but in the latter half of the line, each grand division from the figure 1 to 3 is subdivided into 50 parts; from 3 to 7, into 20 parts each; and from 7 to 10 at the end of the line, each grand division is also subdivided into 20 parts only, on account of the shortness of the spaces.

Being provided with a diagonal scale of 2000 equal parts, which include a length

length equal to the intended length of the line of Numbers, fix the lower edge of it close to the upper edge of the scale intended to be divided, and applying one side of the square to the upper edge of the diagonal scale, and the other side to the beginning of the diagonal divisions, cut the first cross line in the line of Numbers (where the first 1 is to stand) close by that side of the square ; and then, moving the square onward till the same side of it comes to the number of parts among the diagonals which answer in the Table of Gunter's Numbers to the intended subdivisions between 1 and 2, cut these divisions accordingly, in the line of Numbers, close by the side of the square which was set to the parts in the diagonals answering to these subdivisions ; and so on till the whole line be divided.

Thus, the Tables shew, that the figure 2, at the end of the first grand division (marked in the Table 2.00) must

must answer to 301.03 parts found among the diagonals; the division-line for the figure 3 must answer to (or stand even with) 477.12 parts, the division-line for 4 against 602.60 parts; and so on, to the end of the first half of the line, where the second figure 1 stands against 1000. The other half of the line is divided the same way, by the other 1000 diagonal parts of the scale.—The subdivisions which the operator chooses to put into this line must be cut even with the like number of parts found among the diagonals as answer to them in the Table.

In order to divide the lines of Rhumbs, Sines, Verfed Sines, Tangents, and Meridional Parts, on *Gunter's Scale*, the diagonal scale must be placed the contrary way to what it was for dividing the line of Numbers, because all these lines are divided backward, or from the right hand toward the left. Therefore, turn the diagonal

diagonal scale, and placing its contrary edge to the upper edge of the scale to be divided, fix it so, as that, when the square is applied, the beginning of the diagonal divisions shall stand just even with the end of the line of Numbers: and then, applying the cross side of the square to the same number of parts among the diagonals which answer to the degrees, half degrees, &c. in the Tables of Rhumbs, Sines, Versed Sines, Tangents, and Meridional Parts, cut these divisions in the proper lines close by that side of the square.

As Tables of this kind were never all in print before (at least so far as I ever heard of) and they serve not only for dividing the lines accurately on Scales and Sectors, but also for examining and proving whether the Scales and Sectors which are sold at shops be accurately divided or not, I hope they will be acceptable, not only to the makers of these instruments in

general, but also to those who use them.

*How to examine the Divisions of the Lines
on Sectors and Scales.*

If the line of Lines (which is a line of equal parts) on the Sector be accurately divided, which may be easily tried with a pair of compasses, it will serve for examining all the other lines which are drawn from the center of the joint; for all their divisions ought to answer to the equal parts of that line, as they do to the like equal parts of a diagonal scale from which they are supposed to be laid down according to the preceding directions.

Whatever the length of the Sector be, if the line of Chords upon it goes no further than 60 degrees, the line of Lines contains 10 grand divisions, marked 1, 2, 3, 4, 5, 6, 7, 8, 9, 10: but if the Chords go on to 90, the line of Lines

Lines ought to contain 14¹4.2 equal parts, supposing each grand division to be subdivided into 100.

On Sectors six inches long, each grand division of this line is subdivided into 20 equal parts; and if each of these parts be supposed to be subdivided into 5 (which is left to be estimated by the eye) each grand division will contain 100 parts; and consequently the whole ten will contain 1000.

On Sectors 12 inches long, these grand divisions are seldom subdivided into more than 20 parts each, altho' there might very well be 50, and then each part could be easily divided into two by estimation of the eye; and consequently the whole line would contain 1000 equal parts.

This being understood, set one foot of the compasses in the center of the joint, and extending the other foot to the same number of parts in the line of Lines as agree with the tabular

S number

number of parts for any degree of the Chords, Sines, Tangents, or Secants, turn *that* foot to the line of degrees you want to examine; and if it falls into the degrees answering respectively to the tabular numbers, the line is truly divided; otherwise not.

But if the Sector be too long for the points of the compasses to take in the whole length of the line of Lines between them, open it so far, as that the compasses may reach conveniently across the Sector from 10 in the line of Lines on one leg to 10 in the line of Lines on the other: and then, taking the tabular numbers of parts in the compasses across the Sector, in each line of Lines which answer to the tabular number of parts for each degree of the line of Lines, Chords, Tangents, or Secants, apply that extent across the Sector to the like degrees of these lines: and if the points of the compasses fall directly into these graduations, the lines are accurately divided; otherwise not.

To

To examine the lines on plain Scales or Gunter's Scales, open the Sector so, as that the length of the radius of the Chords, or whole length of the line of Lines, taken in the compasses, their points shall then reach from 10 in one line of Lines on the Sector to 10 on the other: and then, taking the tabular numbers for the degrees of the Chords, Sines, or Tangents, across the Sector from one line of Lines to the other, set one foot of the compasses in the beginning of the line to be examined, and the other foot forward among the degrees of that line; and if it falls just into each given degree, the line is truly divided; but falsely if it does not.

Thus you may examine the whole line of Lines, the line of Chords to 60 degrees, and the Tangents to 45: but, in order to examine the degrees of the Tangents above 45, and all the degrees of the Secants, you must take the following method.

The Sector remaining at the same opening as before, take the supplemental tabular numbers in your compasses across the Sector from one line of Lines to the other, which answer to the respective degrees of the Tangents and Secants, and apply that extent across the Sector to the like degrees in these lines: if the compass-points fall exactly into them, the lines are truly graduated.

For Gunter's lines of Numbers, Rhumbs, Sines, Versed Sines, Tangents, and Meridional Parts, set one foot of the compasses in the beginning of the line of Numbers (at the first figure 1) and open the compasses till the other foot falls into the second 1 at the middle of the line. Then open the Sector so, as that the same opening of the compasses shall reach from 10 in the line of Lines on one leg to 10 in the like line on the other. Then, taking the tabular numbers in your compasses across the Sector in these

two last-mentioned lines, which answer to the divisions of the lines you want to examine, apply that extent forward from the left-hand end of the line of Numbers, but backward from the right-hand end of the other lines; and if these divisions or graduations are found by the compasses to agree with the tabular numbers belonging to them, the lines are truly divided; otherwise not.

N. B. In the Sectoral Lines, which proceed from the center of the joint, each has three parallel straight lines drawn for limiting the greater and lesser divisions: in each of these, it is the innermost line to which the points of the compasses must be applied; as that is the only line of the three that proceeds directly from the center of the joint.—I have dwelt the longer on this subject, because these instruments are in the hands of every Mathematician, and it is of the utmost importance to have them rightly divided.

Having shewn how to divide and examine the Lines on Scales and Sectors, it is natural to suppose that the divider would be willing to know, not only how to examine the Tables themselves, but also how to construct them.

The Tables of Natural Sines, Tangents, and Secants, are copied from those in Sherwin's Tables thereof.

The right Sine of an Arc is half the Chord of double that Arc. Therefore, double the Natural Sine of half the given Arc (or number of degrees) and *that* will be the Chord of the whole Arc, or number of degrees required; which, in the Tables, is carried no further than to 90.

The construction of the Table of Natural Rhumbs is the same as that of the Chords.

The Table of Numbers is only the logarithms of these Numbers. The logarithm of unity (or 1) being nothing, is not inserted in the Table: the logarithm of 1.01 (the same as that of

101 without the characteristic) is 43214, which may be put down 4.32; the logarithm of 1.02 is 86002, which may be put down 8.60; and so on, for laying down the line of Numbers on Gunter's Scale.

For the same scale, the Sines are the respective logarithmic Sines subtracted from radius, or the Co-secants less radius.

The Tangents (for the same scale) are the logarithmic Tangents, rejecting radius.

The Versed Sines are double the logarithmic Secants of half the given number of degrees.

For the Meridional Parts, divide the Meridional Parts in any Table thereof by 60.

For the Sine Rhumbs, having found the degrees in every point and quarter-point of the Mariner's Compafs, find the logarithmic Sines thereof, and take their complements arithmetical from radius, or from 8 points.

The same holds good for Tangent Rhumbs, remembering they extend only to 4 points on the scale, and that both the Rhumb and its complement stand on the same point of the line; a logarithm and its reciprocal being equally distant from radius on the opposite sides.

By these rules I have made the foregoing Tables from the now common Tables of Logarithms, Sines, Tangents, Secants, and Versed Sines, with all the care and accuracy that I possibly could, without regarding the time and pains required to construct them.

Of the plain Scale, Sector, and Gunter's Scale.

The lines on the plain Scale are useful in most branches of the mathematics, and its use in each of them is to be found in almost every treatise of the practical mathematics.

The

The Sector is principally useful as a universal plain Scale, fitted to any radius within the compass of its opening; only observing that the equal Parts, Sines, Chords, and Tangents under 45 degrees, are not taken along one line, as on the plain Scale, but across the Sector from any degree on one leg to the same degree of the same line on the other.

The Gunter's lines are for readily working proportions; in which, regard must be had to the terms, whether arithmetical or trigonometrical; that the first and third term may be of the same name, and the second and fourth of the same name likewise: then, raising your proportion according to these rules, take the extent on their proper line, from the first term to the third, in your compasses; and applying one point of the compasses to the second, the other applied to the right or left according as the fourth term is to be more or less than the

the second, will reach to the fourth.
Three examples will explain this.

1.

If 4 yards of cloth
cost 18 shillings,
Then 32 yards
will cost?

2.

As radius
to the hypotenuse 120,
So the sine of the angle opposite the base $30^{\circ} 17'$
to the base.

3.

As the co-sine of the latitude $51^{\circ} 30'$
(= the sine of $38^{\circ} 30'$)
is to radius,
So is the sine of the Sun's declin. $20^{\circ} 14'$
to the sine of the Sun's amplitude.

Note, The Line of Numbers (on Gunter's Scale or on the Sector) is intended to supply the Table of Logarithms, Those of Sines, Tangents, (and Secants, which are found in the same plane with their Co-sines) and the Verfed Sines, supply the places where their logarithms are necessary in calculation.

Now,

Now, in the first question, on the Line of Numbers take in your compasses the distance between 4 and 32; apply one foot thereof on the same line at 18, and the other will reach to 144, the shillings required.

In the second, the distance between radius (or the Sine of 90°) and the Sine of $30^\circ 17'$, taken from the Line of Sines, and one foot applied to the hypothenuse 120 on the Line of Numbers, and the other applied to the left (as the legs of a right lined and right angled triangle are less than the hypothenuse) that foot will reach to $60\frac{1}{2}$, the length of the base required.

The third is wrought wholly on the Line of Sines. The distance between the Sines of $38^\circ 30'$ and $20^\circ 14'$ taken in your compasses, set one foot on the radius or Sine of 90° , and the other will reach to $33\frac{3}{4}$, the Sun's amplitude required.

In the same manner are used the Tangents, Secants, and Versed Sines, in

in proportions where they are required: though sometimes the Versed Sine is taken when the other foot stands on the Line of Sines, as in finding the azimuth, &c. which is easily performed when the art of raising the proportion is known.

A short Account of the Logarithms.

The invention of Logarithms, says the great Dr. Halley, is justly esteemed one of the most useful discoveries in the art of numbers. This assertion is corroborated by the united testimonies of every learned mathematician who has made mention of them, from the time of their first publication to this day.

Lord Napier, of Merchiston in Scotland, in the year 1614, published the first specimen of these useful numbers, under the title of *Mirifici Logarithmorum Canonis descriptio*; a book which was

was received with transport in the mathematical world. And tho' the author had reserved the method of constructing his Table, till the sense of the learned upon his invention should be known ; yet Kepler, Speidell, and others, abroad and at home, laboured at the computation of Logarithms, and constructed small tables thereof, conformable to the plan of Lord Napier.

The description and use of the Canon being in Latin, induced Mr. Edward Wright, a learned mathematician of those times, and to whom we owe the principles of that falsely called Mercator's Sailing, to translate it for the benefit of such of his pupils and others, who not understanding the original, were yet desirous of being acquainted with so valuable a performance. This he effected with great care ; but unhappily dying before the publication, that office devolved to his son, Samuel Wright, who, with the assistance

assistance of Mr. Briggs, then Geometry Professor at Gresham College, and whose name shines with great lustre in a history of Logarithms, published it in the year 1616 or 1618*.

This translation was dedicated by the publisher to the East India Company, who, it seems, had employed Mr. Edward Wright in mathematical affairs: in which dedication he says,

“ his father’s care to make the translation bear a true resemblance of the original was so great, that he procured the *author’s* perusal of it; who, after *great pains taken therein*, gave approbation to it. And it is apparent enough that he (Edward Wright) esteemed it, and intended to have recommended it as a book of more than ordinary worth, especially to seamen. But shortly after he had it returned out of Scotland, it pleased God to call him away,

* My copy is printed at London for Simon Waterson, 1618.

“ before

“ before he could publish it.” Thus we see that the edition of Edward Wright had some advantages over the first edition, from the revival of the inventor, who also wrote a preface to the same; which, as it sets forth the pursuits that led to this discovery, and as the book is very rare, I shall insert in the words of its noble author, together with a specimen and description of the work itself.

“ The Author’s Preface to the *admirable*
“ *Table of Logarithms.*

“ Seeing there is nothing, right
“ well beloved students in the mathe-
“ matics, that is so troublesome to
“ mathematical practice, nor that doth
“ more molest and hinder calculators,
“ than the multiplications, divisions,
“ square and cubical extractions of
“ great numbers, which, besides the
“ tedious expence of time, are for the
“ most part subject to many slippery
“ errors, I began therefore to consider

“ in my mind, by what certain and
“ ready art I might remove those hin-
“ drances; and having thought upon
“ many things to this purpose, I found
“ at length some excellent brief rules,
“ to be treated of (perhaps) hereafter.
“ But amongst all, none more profit-
“ able than this, which, together with
“ the hard and tedious multiplica-
“ tions, divisions and extractions of
“ roots, doth also cast away from the
“ work itself even the very numbers
“ themselves that are to be multi-
“ plied, divided, and resolved into
“ roots, and putteth other numbers
“ in their places, which perform as
“ much as they can do, only by ad-
“ dition and subtraction, division by
“ 2 or division by 3: which secret in-
“ vention being, as all other good
“ things are, so much the better as it
“ shall be the more common, I thought
“ good heretofore to set forth in *Latin*
“ for the public use of mathemati-
“ cians. But now, some of our coun-
“ trymen

“ trymen in this Island, well affected
“ to these studies, and the more pub-
“ lic good, procured a most learned
“ mathematician to translate the same
“ into our vulgar English tongue;
“ who, after he had finished it, sent
“ the copy of it to me, to be seen and
“ considered of by myself. I having
“ most willingly and gladly done the
“ same, find it to be most exact, and
“ precisely conformable to my mind,
“ and the original. Therefore, it may
“ please you who are inclined to these
“ studies, to receive it from me and
“ the translator with as much good-
“ will as we recommend it unto you.
“ Fare ye well.”

And in his dedication to Prince Charles (afterwards King Charles I.) he says, “ This his new invention
“ doth clean take away all the diffi-
“ culty that heretofore hath been in
“ mathematical calculation; and is so
“ fitted to help the weakness of me-
“ mory, that, by means thereof, it is

T “ easy

250 SELECT EXERCISES.

“ easy to resolve more mathematical
“ questions in one hour’s space, than
“ otherwise by that wonted and com-
“ monly received manner of Sines,
“ Tangents, and Secants, can be done
“ even in a whole day.”

The following is a specimen of the
first page of the Canon, in which I
have, however, corrected the typ-
ographical errors.

SELECT EXERCISES.

251

Deg. o.

Min.	Sines.	Logarith.	Differen.	Logar.	Sines.	
0	0	Infinite.	Infinite.	.0	1000000.0	60
1	291	8142568	8142568	.1	1000000.0	59
2	582	7449421	7449421	.2	999999.8	58
3	873	7043956	7043956	.4	999999.6	57
4	1164	6756275	6756274	.7	999999.3	56
5	1454	6533131	6533130	1.1	999998.9	55
6	1745	6350810	6350808	1.6	999998.6	54
7	2036	6196659	6196657	2.2	999998.0	53
8	2327	6063129	6063126	2.8	999997.4	52
9	2618	5945345	5945342	3.5	999996.7	51
10	2909	5839986	5839982	4.3	999995.9	50
11	3220	5744676	5744671	5.2	999995.0	49
12	3491	5657665	5657659	6.2	999994.0	48
13	3781	5577622	5577615	7.3	999992.8	47
14	4072	5503514	5503506	8.4	999991.7	46
15	4363	5434523	5434513	9.6	999990.5	45
16	4654	5369984	5369973	10.9	999989.2	44
17	4945	5309360	5309348	12.3	999987.8	43
18	5236	5252202	5252188	13.8	999986.3	42
19	5527	5198136	5198121	15.4	999984.7	41
20	5818	5146843	5146826	17.0	999983.1	40
21	6109	5098054	5098035	18.7	999981.3	39
22	6399	5051534	5051514	20.5	999979.5	38
23	6690	5007083	5007061	22.4	999977.5	37
24	6981	4964524	4964499	24.5	999975.6	36
25	7272	4923703	4923676	26.5	999973.6	35
26	7563	4884483	4884454	28.7	999971.4	34
27	7854	4846743	4846712	30.9	999969.2	33
28	8145	4810376	4810343	33.2	999966.8	32
29	8436	4775286	4775250	35.5	999964.4	31
30	8726	4741385	4741347	38.1	999961.9	30

Min.

Deg. 89.

T 2

The columns marked Sines are the natural sines answering to the degree and minute of the first and last columns; the adjoining ones their logarithms, and the middle one the differences between the logarithms. The Author explains every one of these columns; but I shall only insert, in his words, what relates to those containing the logarithms.

“ § 11. The 3d column containeth
“ the logarithms of the arches and
“ sines towards the left hand.

“ § 13. And they are also the loga-
“ rithms of the complements of the
“ arches and sines towards the right
“ hand, which we call Antilogarithms.

“ § 14. The fifth column containeth
“ the logarithms of the arches and
“ sines towards the right hand.

“ § 16. They are also the antilogar-
“ rithms of the arches and sines to-
“ wards the left hand, or the loga-
“ rithm of the complements.

“ § 17. Lastly, the 4th or middle
“ column containeth the differences
“ between

“ between the logarithms of the 3d
“ and 5th columns; and so this co-
“ lumn is twofold, abounding and
“ defective.

“ § 18. Those differences are abound-
“ ing which arise out of the subtrac-
“ tion of the logarithms of the fifth
“ column from the logarithms of the
“ third column.

“ § 19. But the differences arising
“ by subtraction of the logarithms of
“ the third column out of the loga-
“ rithms of the fifth column, are de-
“ fective; which therefore are less
“ than nothing.

“ § 20. The abounding differences
“ are called the differential numbers
“ of the arches towards the left hand.

“ § 22. And are also the logarithms
“ of the tangents of the left hand
“ arches.

“ § 23. But the defective differences
“ are called the differential numbers
“ of the right hand arches.

“ § 25. And are also the logarithms

“ of the tangents of the right hand
“ arches.”

And as *admonitions*, amongst others, he gives the following:

“ 28. Here it is to be noted, that
“ if you make the logarithms of the
“ 3d column defective, setting before
“ them this mark —, they shall be
“ made the logarithms of the hypo-
“ thenuses or secants of the right hand
“ arches of the seventh column.

“ 30. And if you make the loga-
“ rithms of the fifth columns defec-
“ tive, they shall be the logarithms of
“ the hypotenuses, or of the secants
“ of the left hand arches of the first
“ column.”

Thus the nature of the Canon required some knowlege of algebraic addition and subtraction; and it was besides troublesome thereby to find the intermediate numbers; the logarithms being only given for such numbers

bers as were equal to the natural sines of each minute. The latter inconvenience E. Wright endeavoured to remove by a scale, of which a draught and description is given in the translation of the admirable Canon, but which Mr. Briggs performed much better in the same work, by means of a subsidiary table of simple application.

The first of these inconveniences was well removed by John Speidell, who was a very ingenious man, and first completed the Canon for trigonometrical use, by adding three other columns, which were the complements arithmetical of Lord Napier's, to wit, Secant, and Co-secant and Co-tangent less radius. By this means he saved the application of the plus and minus, and rendered the calculus of triangles more easy. This he published in the year 1619, under the title of *New Logarithms*; and to the sixth impression thereof in 1624, is

added a Table of the Logarithms of Natural Numbers to 1000, in which the logarithm of 1 is nothing, and that of one thousand, 690775.—Kepler too, in the year 1724, published at Marpurg his *Chilias Logarithmorum ad totidem Numeros rotundos*; in which his logarithm of 1 was 1611809.59, and of 100000, nothing; which also answered to the logarithm of 90°. And as Napier had given the logarithms to every degree and minute of the quadrant, so Kepler gives the nearest corresponding degree and minute, answering to each of his logarithms.

But of all that bestowed their labour on the first sort of logarithms, Benjamin Ursinius is most worthy of mention, who calculated a table of such logarithms to every 10 seconds, which was printed at Cologne in 1625, and to which Vlaac must have been much obliged in constructing his *Canon Magnus*.

These attempts served to facilitate and enlarge the use of Napier's first system

system of logarithms, but were all nevertheless subject, in some measure, to its inconveniencies. The perfection of those noble numbers was reserved as an additional honour to their first inventor; and the present system, than which there can scarcely be hoped for a better, is likewise the invention of Lord Napier.

Mr. Henry Briggs, at the time when the *Canon Mirificus* appeared, was Geometry Professor at Gresham College. This man joined to a great genius and wonderful sagacity in mathematical affairs, a most indefatigable industry, with singular purity of manners, candour, and generosity. Inflamed by this truly admirable work, he writes, on the 10th of March 1615, to Mr. (afterwards Archbishop) Usher, "that
" he was wholly taken up and em-
" ployed about the noble invention of
" logarithms, then lately discovered." And again, "Naper Lord of Markin-
" ston hath set my head and hands at
" work

“ work with his new and admirable logarithms ; I hope to see him this Summer, if it please God ; for I never saw a book which pleased me better, and made me more wonder.”

And we find he actually computed new logarithms, wherein he made o the fine total, as in the Canon, but made 1000000000 the logarithm of one-tenth part of the whole fine, or of 5 degrees 44 minutes 21 seconds, judging them more commodious than those of Napier ; which, in the Summer of 1616, he carried with him to Edinburgh, where he was received by the Baron with great kindness, and staid with him a month.

It was in the course of this time, by Mr. Briggs's account, that Lord Napier communicated to him his intended alteration of the Canon ; and that he had caused it to be printed in its present form, till his health and leisure should permit him to calculate another, wherein he intended to make

o the

o the logarithm of unity, and 10 that of the sine total. Briggs, greatly pleased with that suggestion, which he confessed was much more eligible than his own, threw aside those he had begun, and, as well from his own inclination as the earnest entreaty of Lord Napier, on his return to London, set about the calculation of new ones, after that form: of which having computed 1000 to 8 places of figures, besides the index, he carried them, in the Summer of 1617, to the Baron, who was highly pleased therewith, and pressed his beloved friend Briggs, as he stiles him, to continue the work. Which injunction he afterwards more strongly enforced, by the public testimonies he gave of both the capacity and industry of his friend; and the expectations the world might form from his continuing the great work, of which his Lordship, in regard to these improved logarithms, challenged little more than the invention and mode of construction.

Nor were the sentiments of Briggs in regard to Lord Napier less lively than those of the latter for him; he mentions him every where with the utmost veneration, warmth, and affection; and seems perfectly enraptured by the nobleness and utility of the great invention. And indeed nothing less could have sustained him through the immense labour of such computations as he underwent in bringing it into practice.

On his return to London, he printed the *Chilias Prima*; and seven years after, in 1624, he produced his *Arithmetica Logarithmica*, wherein he gives the logarithms of 31000 natural numbers to 14 places of figures, besides the index; a work which will appear stupendous to any one who, by the same method, will take the trouble to compute the logarithms of only two or three such numbers; which method he may find in the said work, in Malcolm's Arithmetic, Keil's Euclid, Ward's Mathematics, and several other writers;

writers; and of which an idea may be formed from the account of Dr. Halley, who says, that to have his logarithm true to 14 places of figures, Mr. Briggs, by continual extraction, was obliged to find the root of more than the 140 million of millioneth power. Euclid Speidell says, he was told, that only finding the logarithm of 2 true to 15 places, for the aforesaid Table, was the work of eight persons a whole year; and Pardies, in his Geometry, praises God, who, for the public good, has roused up persons to whom he had given sufficient patience to surmount the fatigue of a labour seemingly insupportable: "for," says he, "we know that more than 20 persons, employed for that purpose, spent upwards of 20 years with indefatigable assiduity in the calculation." This last testimony, which some have thought an insinuation that the logarithms were first invented in France, rather proves the genius and

ability of our excellent countryman Briggs, who, in eight years, with scarce as many persons, performed a much greater work than 20 Frenchmen in thrice the time: for the largest French tables of those times were carried to only 11 places of figures, and these are supposed to have been taken from Vlaac.

Though the *Chilias Prima* was printed in the latter end of the year 1617, it was nevertheless not published till after the death of Napier, which happened on the 3d of April, 1618; for, in the preface thereto, Briggs says, “ why these logarithms differ from those set forth by their most illustrious inventor, of ever worshipful memory, in his *Canon Mirificus*, it is to be hoped his posthumous book will shortly make appear.”

In 1620, two years after the *Chilias Prima* of Briggs was published, Mr. Edmund Gunter set forth a Canon of Sines and Tangents, adapted to these loga-

logarithms, which was the first of that kind, and to which he gave the name of *Artificial Sines and Tangents*. This must have cost Gunter some labour, though he appears to have been much beholden to the *Chilias*, which consists of 8, and these only of 7 places, besides the index, of which the last place is anomalous, when compared with those which Briggs and Vlaac afterwards computed. These he republished in his book *De Sectore et Radio*, in 1623, together with the *Chilias Prima* of his old colleague.

But Briggs himself lived to complete a Table of logarithmic Sines and Tangents, to the one hundredth part of every degree, and to 14 places of figures, besides the index; to which he intended to have written a full description and use, when death put an end to his labours, at the age of 74, on the 26th January, 1637, thirteen years after his beloved Napier.

It is remarkable of these two great men, that they both undertook these mighty

mighty labours at rather an advanced age. The Compendium of Honour mentions Napier to have died in his 67th year; and as it is generally supposed the logarithms were invented by him about 1610, he must have been near 60 years of age, and, as himself informs us, weak and infirm of body, when he began the calculation of his Canon; and in 1614, when Briggs first applied his thoughts that way, he must have been upwards of 57: which may teach us, there is no time of our life too late to acquire honest fame, when we are obedient to the voice of genius, and keep our ease in due subjection to industry.

Briggs, when dying, recommended his last-mentioned work to the care and completion of Henry Gellibrand, then Astronomy Professor at Gresham College, who added thereto the description and use of the Canon, and published it at London*, in 1633, un-

* At Gouda, according to some writers.

der the title of *Trigonometria Britannica*.

Thus were these numbers completely planned by the wisdom of the ingenious Lord of Merchiston, and the great building rear'd, in nice conformance thereto, by the most industrious and learned Henry Briggs, in a few years; and it is difficult to determine in these two great men, which is the most admirable; the sagacity of the inventor, or the indefatigable application of the calculator.

In the *Arithmetica Logarithmica*, Mr. Briggs had calculated the logarithms of all the natural numbers from 1 to 20000, and from 90000 to 101000, leaving the interval to be filled by the ingenious; to any of whom, in his preface, he offers paper properly ruled, and necessary instructions, he purposing, in the mean time, to employ himself on his Triangular Canon, and having left no labour which an ordinary skill might not perform. But Adrian Vlaac, of Targou, or Gouda

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in Holland, completed the 70000 intermediate numbers with such expedition, that, in 1628, he published the second edition of the *Arithmetica Logarithmica*, wherein was contained all the logarithms from 1 to 100000 to 10 places of figures, together with a Table of *Artificial Sines, Tangents, and Secants* (as Gunter had called them) to every minute of the quadrant.

Considering also that the usual method of angular supputation was by minutes and seconds, Vlaac, in the same year with the *Trigonometria Britannica*, published his *Trigonometria Artificialis*; wherein is contained the logarithmic Sines and Tangents to every 10 seconds, and the logarithms from 1 to 20000: which shews Vlaac to have been a very assiduous and ingenious man; though Dr. Newton, Norwood, and also Wingate, seem to censure both him and others, who rashly published new editions of Briggs's *Logarithmica Arithmetica* without his leave; thereby preventing the additions he intended

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intended to give it when republished, and for making mutilated translations of that and his *Trigonometria Britannica*, to the discredit as well as disadvantage of the author. Be this as it will, Vlaac's editions are in great esteem, for having the intermediate Chiliads, for his Sexagenary Notation, and for the exactitude of his Tables.

These Tables have been reduced into a more compendious form by the Reverend Nathaniel Roe, of Binacre in Suffolk, in 1633; by Dr. Newton in 1658; and the finishing hand seems to be given to them, in point of disposition and fulness, by Sherwin, in 1706; in whose Mathematical Tables are contained, to 7 places of figures besides the index, the logarithms of numbers from 1 to 100000, with natural and logarithmical Sines, Tangents, Secants, and Versed Sines, to every minute of the quadrant.

The honour of this noble and useful invention has been almost universally attributed to the Great Man to whom

it is due ; yet there have been some found who have, through envy, prejudice, or want of consideration, attempted to lessen or transplant it. And, first, Wood, in his *Athenæ Oxoniensis*, tells it as a report of Oughtred's, that one Dr. Craig first gave Lord Nepair an intimation that such numbers were invented by Longomantanus ; which set him about calculating his first Canon. Now this story refutes itself ; first, by Oughtred's own acknowledgment of Lord Nepair as the inventor ; and by Longomantanus never claiming the invention, though he lived many years after the honour thereof was attributed to Nepair. Wingate, who carried this invention into France, in his publications, asserts, " the second sort of logarithms " were jointly schemed by Briggs and " Nepair ;" contrary to the sense of the very passage he quotes from Briggs himself. And, I presume, following, or rather willing to improve upon him, Saunderson, in his *Algebra*, asserts,

serts, “ that Briggs was *undoubtedly* the “ first that thought of this (the pre-“ sent) system :” which assertion could be contradicted by Lord Napier’s claim in Wright’s Translation of his Canon, and that of his son for him, in the preface to his father’s Posthumous Works, 1619, both printed under the inspection of Briggs, if even the positive testimony of Briggs himself were not against it.

And it may be observed of this ingenious man, that every attempt to lessen his reputation has only served to establish or increase it: the trivial, and often ridiculous, arguments of his opponents; the testimonies of his honest and candid coadjutors Briggs and Wright, as well as of those best acquainted with mathematical history, foreigners as well as Britons, uniting to fix the invention of *Logarithms* in their first and present form to Lord John Napier of Merchiston.

The logarithmic Canon serves to find readily the logarithm of any as-
2 signed

signed number; and we are told by Dr. Wallis, in the 2d volume of his mathematical works, that an antilogarithmic Canon, or one to find as readily the number corresponding to every logarithm, was begun by Mr. Harriot the algebraist (who died in 1621) and completed by Mr. Walter Warner, the editor of Harriot's works, before 1640; which ingenious performance it seems was lost, for want of encouragement to publish it.

A small specimen of such numbers was published in the Philosophical Transactions, for the year 1714, by Mr. Long of Oxford; but it was not till 1742 that a complete antilogarithmic Canon was published, by Mr. James Dodson, wherein he has computed the numbers corresponding to every logarithm from 1 to 100000, to eleven places of figures.

The logarithmic numbers were disposed on straight lines by *Gunter* in 1623, and hence the Gunter's Scale: by *Wingate*, 1627; on two rulers sliding against

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against each other, to save the use of compasses in working, and also on circles, by *Oughtred*, about 1627; in a special form by Mr. *Milburne* of Yorkshire, about 1650; and lastly, on the present sliding Gunter, by *Seth Partridge*, in 1657.

The errors of the press are never more numerous than in books of Mathematics, especially Tables; the calculist, compositor, examiner, and even the printer, all being liable to produce them. Warmth and precipitancy are the means which occasion them in the former; and the printer's ink-balls will sometimes pull out a letter or figure, which he has not always the knowledge of, and when he has, does not perhaps take the pains to repair the injury properly: a figure snatched up in haste, often supplying, at a venture, the deficiency. From hence it seems next to impossible to be assured that a whole impression is alike, or that it is free from error. Now, as the present series of logarithms requires

quires nothing but correctness to make it perfect, I cannot help concluding this short discourse with a wish, that our public-spirited noblemen and gentlemen would unite in producing a national edition of this most excellent BRITISH INVENTION, in a manner worthy themselves.

If Gardiner's Logarithms, freed from errors, were deeply engraved on strong copper-plates, they would endure for ages ; would be free from typographical *lacunæ* ; and, as the errors occasionally found therein might be corrected by the engraver, would be continually increasing in value. Also, as by national or private generosity, in defraying part of the first expence, the work might be sold reasonable enough to come within the purchase of most mathematicians, it would be a universal benefit, and will immortalize his name who first promotes it.

A SHORT
A C C O U N T
O F T H E
LIFE OF THE AUTHOR.

AS this is probably the last Book I shall ever publish, I beg leave to prefix to it a short account of myself, and of the manner I first began, and have since prosecuted my studies. For, as my setting out in life from a very low station, and in a remote part of the Island, has occasioned some false and indeed very improbable particulars to be related of me, I therefore think it the better way, instead of contradicting them one by one, to give a faithful and circumstantial detail of my whole proceedings, from my first

a obscure

obscure beginning to the present time: wherein, if I should insert some particulars of little moment, I hope the good-natur'd Reader will kindly excuse me.

I was born in the year 1710, a few miles from Keith, a little village in Bamffshire, in the North of Scotland; and can with pleasure say, that my parents, though poor, were religious and honest; lived in good repute with all who knew them, and died with good characters.

As my father had nothing to support a large family but his daily labour, and the profits arising from a few acres of land which he rented, it was not to be expected that he could bestow much on the education of his children: yet they were not neglected; for, at his leisure hours, he taught them to read and write. And it was while he was teaching my elder brother to read the Scotch Catechism that I acquired my reading.

Ashamed

LIFE OF THE AUTHOR. iii

Ashamed to ask my father to instruct me, I used, when he and my brother were abroad, to take the Catechism, and study the lesson which he had been teaching my brother: and when any difficulty occurred, I went to a neighbouring old woman, who gave me such help as enabled me to read tolerably well before my father had thought of teaching me.

Some time after, he was agreeably surprised to find me reading by myself: he thereupon gave me further instruction, and also taught me to write; which, with about three months I afterward had at the grammar-school at Keith, was all the education I ever received.

My taste for mechanics arose from an odd accident.—When about 7 or 8 years of age, a part of the roof of the house being decayed, my father, desirous of mending it, applied a prop and lever to an upright spar to raise it to its former situation; and, to my

great astonishment, I saw him, without considering the reason, lift up the ponderous roof as if it had been a small weight. I attributed this at first to a degree of strength that excited my terror as well as wonder: but thinking further of the matter, I recollect-
ed that he had applied his strength to that end of the lever which was furthest from the prop; and finding, on enquiry, that this was the means whereby the seeming wonder was ef-
fected, I begun making levers (which I then called bars); and by applying weights to them different ways, I found the power gained by my bar was just in proportion to the lengths of the different parts of the bar on ei-
ther side of the prop.—I then thought it was great pity that, by means of this bar, a weight could be raised but a very little way. On this, I soon imagined, that, by pulling round a wheel, the weight might be raised to any height by tying a rope to the

weight, and winding the rope round the axle of the wheel ; and that the power gained must be just as great as the wheel was broader than the axle was thick ; and found it to be exactly so, by hanging one weight to a rope put round the wheel, and another to the rope that coiled round the axle. So that, in these two machines, it appeared very plain, that their advantage was as great as the space gone thro' by the working power exceeded the space gone through by the weight : and this property I also thought must take place in a wedge for cleaving wood ; but then, I happened not to think of the screw.—By means of a turning lathe which my father had, and sometimes used, and a little knife, I was enabled to make wheels and other things necessary for my purpose.

I then wrote a short account of these machines, and sketched out figures of them with a pen, imagining it to be the first treatise of the kind that ever

was written: but found my mistake, when I afterward shewed it to a gentleman, who told me that these things were known long before, and shewed me a printed book in which they were treated of: and I was much pleased when I found, that my account (so far as I had carried it) agreed with the principles of mechanics in the book he shewed me. And from that time my mind preserved a constant tendency to improve in that science.

But, as my father could not afford to maintain me while I was in pursuit only of these matters, and I was rather too young and weak for hard labour, he put me out to a neighbour to keep sheep, which I continued to do for some years; and in that time I began to study the stars in the night. In the day-time I amused myself by making models of mills, spinning-wheels, and such other things as I happened to see.

I then went to serve a considerable farmer in the neighbourhood, whose name

name was James Glashan. I found him very kind and indulgent; but he soon observed, that in the evenings, when my work was over, I went into a field with a blanket about me; lay down on my back, and stretched a thread with small beads upon it, at arms' length, between my eye and the stars; sliding the beads upon it till they hid such and such stars from my eye, in order to take their apparent distances from one another; and then, laying the thread down on a paper, I marked the stars thereon by the beads, according to their respective positions, having a candle by me. My master at first laughed at me; but, when I explained my meaning to him, he encouraged me to go on: and that I might make fair copies in the day-time of what I had done in the night, he often worked for me himself. I shall always have a respect for the memory of that man.

One day he happened to send me with a message to the Reverend Mr.

John Gilchrist, minister at Keith, to whom I had been known from my childhood. I carried my star-papers to shew them to him, and found him looking over a large parcel of maps, which I surveyed with great pleasure, as they were the first I had ever seen. He then told me that the Earth is round like a ball, and explained the map of it to me. I requested him to lend me *that* map, to take a copy of it in the evenings. He chearfully consented to this, giving me at the same time a pair of compasses, a ruler, pens, ink, and paper; and dismissed me with an injunction not to neglect my master's business by copying the map, which I might keep as long as I pleased.

For this pleasant employment, my master gave me more time than I could reasonably expect; and often took the threshing-flail out of my hands, and work'd himself, while I sat by him in the barn, busy with my compasses, ruler, and pen.

When

When I had finished the copy, I asked leave to carry home the map: he told me I was at liberty to do so, and might stay two hours to converse with the minister.—In my way thither, I happened to pass by the school at which I had been before, and saw a genteel-looking man (whose name I afterward learnt was Cantley) painting a sun-dial on the wall. I stopt a while to observe him, and the school-master came out, and asked me what parcel it was that I had under my arm. I shewed him the map, and the copy I had made of it, wherewith he appeared to be very well pleased, and asked me whether I should not like to learn of Mr. Cantley to make sun-dials. Mr. Cantley looked at the copy of the map, and commended it much; telling the school-master (Mr. John Skinner) that it was a pity I did not meet with notice and encouragement. I had a good deal of conversation with him, and found him to be quite affable

ble and communicative; which made me think I should be extremely happy if I could be further acquainted with him.

I then proceeded with the map to the minister, and shewed him the copy of it.—While we were conversing together, a neighbouring gentleman, Thomas Grant, Esq; of Achoynaney, happened to come in; and the minister immediately introduced me to him, shewing him what I had done. He expressed great satisfaction, asked me some questions about the construction of maps, and told me, that if I would go and live at his house, he would order his butler, Alexander Cantley, to give me a great deal of instruction. Finding that this Cantley was the man whom I had seen painting the sun-dial, and of whom I had already conceived a very high opinion, I told 'Squire Grant, that I should rejoice to be at his house as soon as the time was expired for which I was engaged

gaged with my present master.—He very politely offered to put one in my place ; but this I declined.

When the term of my servitude was out, I left my good master, and went to the gentleman's house, where I quickly found myself with a most humane good family. Mr. Cantley the butler soon became my friend, and continued so till his death. He was the most extraordinary man that I ever was acquainted with, or perhaps ever shall see ; for he was a complete master of arithmetic, a good mathematician, a master of music on every known instrument except the harp, understood Latin, French, and Greek, let blood extremely well, and could even prescribe as a physician upon any urgent occasion. He was what is generally called *self-taught* ; but, I think, he might with much greater propriety have been termed GOD ALMIGHTY's scholar.

He immediately began to teach me decimal arithmetic, and algebra ; for I had

I had already learnt vulgar arithmetic, at my leisure hours, from books. He then proceeded to teach me the elements of geometry; but, to my inexpressible grief, just as I was beginning that branch of science, he left Mr. Grant, and went to the late Earl of Fife's, at several miles distance. The good family I was then with could not prevail with me to stay after he was gone; so I left them, and went to my father's.

He had made me a present of Gordon's Geographical Grammar, which, at that time, was to me a great treasure. There is no figure of a globe in it, although it contains a tolerable description of the globes, and their use. From this description I made a globe in three weeks at my father's, having turned the ball thereof out of a piece of wood; which ball I covered with paper, and delineated a map of the world upon it; made the meridian ring and horizon of wood; covered them with paper, and graduated them;

them; and was happy to find, that, by my globe (which was the first I ever saw) I could solve the problems.

But this was not likely to afford me bread, and I could not think of staying with my father, who I knew full well could not maintain me in that way, as it could be of no service to him; and he had, without my assistance, hands sufficient for all his work.

I then went to a miller, thinking it would be a very easy business to attend the mill, and that I should have a great deal of leisure-time to study decimal arithmetic and geometry. But my master, being too fond of tippling at an ale-house, left the whole care of the mill to me, and almost starved me for want of victuals; so that I was glad when I could have a little oat-meal mixed with cold water to eat. I was engaged for a year in that man's service, at the end of which I left him, and returned in a very weak state to my father's.

Soon

Soon after I had recovered my former strength, a neighbouring farmer, who practised as a physician in that part of the country, came to my father's, wanting to have me as a labouring servant. My father advised me to go to Doctor Young, telling me that the Doctor would instruct me in that part of his business. This he promised to do, which was a temptation to me. But instead of performing his promise, he kept me constantly to very hard labour, and never once shewed me one of his books. All his servants complained that he was the hardest master they had ever lived with; and it was my misfortune to be engaged with him for half a year. But, at the end of three months, I was so much overwrought, that I was almost disabled; which obliged me to leave him: and he was so unjust as to give me nothing at all for the time I had been with him, because I did not complete my

half-year's service; though he knew that I was not able, and had seen me working for the last fortnight, as much as possible, with one hand and arm, when I could not lift the other from my side. And what I thought was particularly hard, he never once tried to give me the least relief, further than once bleeding me, which rather did me hurt than good, as I was very weak, and much emaciated. I then went to my father's, where I was confined for two months on account of my hurt, and despaired of ever recovering the use of my left arm. And during all that time, the Doctor never once came to see me, although the distance was not quite two miles.— But my friend Mr. Cantley hearing of my misfortune, at twelve miles distance, sent me proper medicines and applications, by means of which I recovered the use of my arm; but found myself too weak to think of going into service again, and had entirely

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lost my appetite, so that I could take nothing but a draught of milk once a-day, for many weeks.

In order to amuse myself in this low state, I made a wooden clock, the frame of which was also of wood ; and it kept time pretty well. The bell, on which the hammer struck the hours, was the neck of a broken bottle.

Having then no idea how any time-keeper could go but by a weight and a line, I wondered how a watch could go in all positions ; and was sorry that I had never thought of asking Mr. Cantley, who could very easily have informed me. But happening one day to see a gentleman ride by my father's house, (which was close by a public road) I asked him what o'clock it then was : he looked at his watch, and told me. As he did that with so much good-nature, I begged of him to shew me the inside of his watch : and though he was an entire stranger, he immediately opened the watch, and put

put it into my hands. I saw the spring-box with part of the chain round it, and asked him what it was that made the box turn round: he told me that it was turned round by a steel spring within it. Having then never seen any other spring than that of my father's gun-lock, I asked how a spring within a box could turn the box so often round as to wind all the chain upon it. He answered, that the spring was long and thin; that one end of it was fastened to the axis of the box, and the other end to the inside of the box; that the axis was fixed, and the box was loose upon it. I told him I did not yet thoroughly understand the matter: Well, my lad, says he, take a long thin piece of whalebone, hold one end of it fast between your finger and thumb, and wind it round your finger: it will then endeavour to unwind itself; and if you fix the other end of it to the inside of a small hoop, and leave it to itself, it will turn the

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hoop round and round, and wind up a thread tied to the outside of the hoop. —I thanked the gentleman, and told him that I understood the thing very well. I then tried to make a watch with wooden wheels, and made the spring of whalebone ; but found that I could not make the watch go when the balance was put on, because the teeth of the wheels were rather too weak to bear the force of a spring sufficient to move the balance ; altho' the wheels would run fast enough when the balance was taken off. I inclosed the whole in a wooden case, very little bigger than a breakfast tea-cup : but a clumsy neighbour one day looking at my watch, happened to let it fall ; and turning hastily about to pick it up, set his foot upon it, and crushed it all to pieces ; which so provoked my father, that he was almost ready to beat the man ; and disengaged me so much, that I never attempted to make such another machine

chine again, especially as I was thoroughly convinced I could never make one that would be of any real use.

As soon as I was able to go abroad, I carried my globe, clock, and copies of some other maps besides that of the world, to the late Sir James Dunbar of Durn (about seven miles from where my father lived) as I had heard that Sir James was a very good-natur'd, friendly, inquisitive gentleman. He received me in a very kind manner, was pleased with what I shewed him, and desired I would clean his clocks. This, for the first time, I attempted; and then begun to pick up some money in that way about the country, making Sir James's house my home, at his desire.

Two large globular stones stood on the top of his gate: on one of them I painted (with oil colours) a map of the terrestrial globe, and on the other a map of the celestial, from a planisphere of the stars which I copied on

paper from a celestial globe belonging to a neighbouring gentleman. The poles of the painted globes stood toward the poles of the heavens ; on each, the 24 hours were placed around the equinoctial, so as to shew the time of the day when the sun shone out, by the boundary where the half of the globe at any time enlightened by the sun was parted from the other half in the shade ; the enlightened parts of the terrestrial globe answering to the like enlightened parts of the earth at all times. So that, whenever the sun shone on the globe, one might see to what places the sun was then rising, to what places it was setting, and all the places where it was then day or night, throughout the earth.

During the time I was at Sir James's hospitable house, his sister, the Honourable the Lady Dipple, came there on a visit, and Sir James introduced me to her. She asked me whether I could draw patterns for needle-work on

on aprons and gowns. On shewing me some, I undertook the work, and drew several for her; some of which were copied from her patterns, and the rest I did according to my own fancy. On this, I was sent for by other ladies in the country, and begun to think myself growing very rich by the money I got for such drawings; out of which I had the pleasure of occasionally supplying the wants of my poor father.

Yet all this while I could not leave off star-gazing in the nights, and taking the places of the planets among the stars by my above-mentioned thread. By this, I could observe how the planets changed their places among the stars, and delineated their paths on the celestial map, which I had copied from the above-mentioned celestial globe.

By observing what constellations the Ecliptic passed through in that map, and comparing these with the starry

heaven, I was so impress'd as sometimes to imagine that I saw the Ecliptic in the heaven, among the stars, like a broad circular road for the sun's apparent course; and fancied the paths of the planets to resemble the narrow rutts made by cart-wheels, sometimes on one side of a plain road and sometimes on the other, crossing the road at small angles, but never going far from either side of it.

Sir James's house was full of pictures and prints, several of which I copied with pen and ink: this made him think I might become a painter.

Lady Dipple had been but a few weeks there, when William Baird, Esq; of Auchmedden, came on a visit: he was the husband of one of that lady's daughters, and I found him to be very ingenious and communicative; he invited me to go to his house and stay some time with him, telling me that I should have free access to his library, which was a very large one; and that

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he would furnish me with all sorts of implements for drawing. I went thither, and staid about eight months; but was much disappointed in finding no books of astronomy in his library, except what was in the two volumes of Harris's Lexicon Technicum, altho' there were many books on geography and other sciences: several of these indeed were in Latin, and more in French; which being languages that I did not understand, I had recourse to him for what I wanted to know of these subjects, which he chearfully read to me; and it was as easy for him, at sight, to read English from a Greek, Latin, or French book, as from an English one. He furnished me with pencils and Indian ink, shewing me how to draw with them: and although he had but an indifferent hand at that work, yet he was a very acute judge; and consequently a very fit person for shewing me how to correct my own work. He was the first

who ever sat to me for a picture, and I found it was much easier to draw from the life than from any picture whatever, as nature was more striking than any imitation of it.

Lady Dipple came to his house in about half a year after I went thither. And as they thought I had a genius for painting, they consulted together about what might be the best way to put me forward. Mr. Baird thought it would be no difficult matter to make a collection for me among the neighbouring gentlemen, to put me to a painter at Edinburgh: but he found, upon trial, that nothing worth the while could be done among them. And as to himself, he could not do much that way, because he had but a small estate, and a very numerous family.

Lady Dipple then told me that she was to go to Edinburgh next Spring, and that if I would go thither, she would give me a year's bed and board

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at her house *gratis*, and make all the interest she could for me among her acquaintance there.—I thankfully accepted of her kind offer; and instead of giving me one year, she gave me two. I carried with me a letter of recommendation from the Lord Pitfligo (a near neighbour of 'Squire Baird's) to Mr. John Alexander, a painter in Edinburgh; who allowed me to pass an hour every day at his house, for a month, to copy from his drawings; and said he would teach me to paint in oil-colours, if I would serve him seven years, and my friends would maintain me all that time: but this was too much for me to desire them to do; nor did I chuse to serve so long. I was then recommended to other painters, but they would do nothing without money. So I was quite at a loss what to do.

In a few days after this, I received a letter of recommendation from my good friend 'Squire Baird to the Rev-
rend

rend Dr. Robert Keith at Edinburgh, to whom I gave an account of my bad success among the painters there. He told me, that if I would copy from nature, I might do without their assistance; as all the rules for drawing signified but very little when one came to draw from the life: and, by what he had seen of my drawings brought from the North, he judged I might succeed very well in drawing pictures from the life, in Indian ink, on vellum. He then sat to me for his own picture, and sent me with it and a letter of recommendation to the Right Honourable the Lady Jane Douglas, who lived with her mother, the Marchioness of Douglas, at Merchiston-house, near Edinburgh. Both the Marchioness and Lady Jane behaved to me in the most friendly manner, on Dr. Keith's account, and sat for their pictures; telling me at the same time, that I was in the very room in which Lord Napier invented and computed

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the Logarithms; and that, if I thought it would inspire me, I should always have the same room whenever I came to Merchiston.—I staid there several days, and drew several pictures of Lady Jane; of whom it was hard to say, whether the greatness of her beauty, or the goodness of her temper and dispositions, was the most predominant. She sent these pictures to ladies of her acquaintance, in order to recommend me to them; by which means I soon had as much business as I could possibly manage, so as not only to put a good deal of money in my own pocket, but also to spare what was sufficient to help to supply my father and mother in their old age.— Thus, a business was providentially put into my hands, which I followed for six and twenty years.

Lady Dipple, being a woman of the strictest piety, kept a watchful eye over me at first, and made me give her an exact account at night of what families

families I had been in throughout the day, and of the money I had received. She took the money each night, desiring I would keep an account of what I had put into her hands ; telling me that I should duly have, out of it, what I wanted for clothes, and to send to my father.—But, in less than half a year, she told me that she would thenceforth trust me with being my own banker ; for she had made a good deal of private enquiry how I had behaved when I was out of her sight through the day ; and was satisfied with my conduct.

During my two years' stay at Edinburgh, I somehow took a violent inclination to study anatomy, surgery, and physic, all from reading of books, and conversing with gentlemen, on these subjects ; which, for that time, put all thoughts of astronomy out of my mind, and I had no inclination to become acquainted with any one there who taught either mathematics or astro-

astronomy: for nothing would serve me but to be a Doctor.

At the end of the second year I left Edinburgh, and went to see my father, thinking myself tolerably well qualified to be a physician in that part of the country; and I carried a good deal of medicines, plaisters, &c. thither.—But, to my mortification, I soon found that all my medical theories and study were of little use in practice. And then, finding that very few paid me for the medicines they had, and that I was far from being so successful as I could wish, I quite left off that business, and began to think of taking to the more sure one of drawing pictures again.—For this purpose I went to Inverness, where I had eight months business.

When I was there, I began to think of astronomy again; and was heartily sorry for having quite neglected it at Edinburgh, where I might have improved my knowlege by conversing with

with those who were very able to assist me.—I began to compare the Ecliptic with its twelve signs (through which the Sun goes in twelve months) to the circle of 12 hours on the dial-plate of a watch, the hour-hand to the Sun, and the minute-hand to the Moon, moving in the Ecliptic; the one always overtaking the other at a place forwarder than it did at their last conjunction before. On this, I contrived and finished a scheme on paper for shewing the motions and places of the Sun and Moon in the Ecliptic on each day of the year, perpetually; and consequently, the days of all the New and Full Moons.

To this I wanted to add a method for shewing the Eclipses of the Sun and Moon; of which I knew the cause long before, by having observed that the Moon was, for one half of her period, on the North side of the Ecliptic, and for the other half on the South. But, having not observed her course

course long enough among the Stars by my above-mentioned thread, so as to delineate her path on my celestial map, in order to find the two opposite points of the Ecliptic in which her orbit crosses it, I was altogether at a loss how and where in the Ecliptic (in my scheme) to place these intersecting points: this was in the year 1739.

At last, I recollect'd, that when I was with 'Squire Grant of Achoynaney in the year 1730, I had read, that on the 1st of January 1690, the Moon's ascending Node was in the 10th minute of the first degree of Aries; and that her Nodes moved backward thro' the whole Ecliptic in 18 years and 224 days, which was at the rate of 3 min. 11 sec. every 24 hours. But, as I scarce knew in the year 1730 what the Moon's Nodes meant, I took no further notice of it at that time.

However, in the year 1739, I set to work at Inverness; and after a tedious calculation of the slow motion of the Nodes

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Nodes from Jan. 1690 to Jan. 1740, it appeared to me, that (if I was sure I had remembered right) the Moon's ascending Node must be in 23 deg. 25 min. of Cancer at the beginning of the year 1740. And so I added the Eclipse-part to my scheme, and called it *The Astronomical Rotula*.

When I had finished it, I shewed it to the Reverend Mr. Alexander Mac-Bean, one of the ministers at Inverness, who told me he had a set of almanacks by him for several years past, and would examine it by the Eclipses mentioned in them. We examined it together, and found that it agreed throughout with the days of all the New and Full Moons and Eclipses mentioned in these almanacks; which made me think I had constructed it upon true astronomical principles. On this, Mr. MacBean desired me to write to Mr. MacLaurin, professor of the mathematics at Edinburgh, and give him an account of the me-

thods by which I had formed my plan, requesting him to correct it where it was wrong. He returned me a most polite and friendly answer (although I had never seen him during my stay at Edinburgh) and informed me that I had only mistaken the radical mean place of the ascending Node by a quarter of a degree; and that, if I would send the drawing of my Rotula to him, he would examine it, and endeavour to procure me a subscription to defray the charges of engraving it on copper-plates, if I chose to publish it. I then made a new and correct drawing of it, and sent it to him, who soon got me a very handsome subscription by setting the example himself, and sending subscription-papers to others.

I then returned to Edinburgh, and had the Rotula-plates engraved there by Mr. Cooper*. It has gone through

* Cooper was master to the justly celebrated Mr. Robert Strange, who was at that time his apprentice.

several impressions, and always sold very well till the year 1752, when the stile was changed, which rendered it quite useless.—Mr. MacLaurin received me with the greatest civility when I first went to see him at Edinburgh. He then became an exceeding good friend to me, and continued so till his death.

One day I requested him to shew me his Orrery, which he immediately did. I was greatly delighted with the motions of the Earth and Moon in it, and would gladly have seen the wheel-work, which was concealed in a brass box, and the box and planets above it were surrounded by an armillary sphere. But he told me, that he never had opened it; and I could easily perceive that it could not be opened but by the hand of some ingenious clock-maker, and not without a great deal of time and trouble.

After a good deal of thinking, and calculation, I found that I could contrive

trive the wheel-work for turning the planets in such a machine, and giving them their progressive motions; but should be very well satisfied if I could make an Orrery to shew the motions of the Earth and Moon, and of the Sun round its axis. I then employed a turner to make me a sufficient number of wheels and axles, according to patterns which I gave him in drawing: and after having cut the teeth in the wheels by a knife, and put the whole together, I found that it answered all my expectations. It shewed the Sun's motion round his axis, the diurnal and annual motions of the Earth on its inclined axis, which kept its parallelism in its whole course round the Sun; the motions and phases of the Moon, with the retrograde motion of the Nodes of her orbit; and consequently, all the variety of seasons, the different lengths of days and nights, the days of the New and Full Moons, and Eclipses.

When it was all completed, except the box that covers the wheels, I shewed it to Mr. MacLaurin, who commended it in presence of a great many young gentlemen who attended his lectures. He desired me to read them a lecture on it, which I did without any hesitation, seeing I had no reason to be afraid of speaking before a great and good man who was my friend.—Soon after that, I sent it in a present to the Reverend and ingenuous Mr. Alexander Irvine, one of the ministers at Elgin in Scotland.

I then made a smaller and neater Orrery, of which all the wheels were of ivory, and I cut the teeth in them with a file.—This was done in the beginning of the year 1743; and, in May that year, I brought it with me to London, where it was soon after bought by Sir Dudley Rider. I have made six Orreries since that time, and there are not any two of them in which the wheel-work is alike: for I could never bear

bear to copy one thing of that kind from another, because I still saw there was great room for improvements.

I had a letter of recommendation from Mr. Baron Edlin at Edinburgh to the Right Honourable Stephen Poyntz, Esq; at St. James's, who had been preceptor to his Royal Highness the late Duke of Cumberland, and was well known to be possessed of all the good qualities that can adorn a human mind.—To me, his goodness was really beyond my power of expression; and I had not been a month in London till he informed me that he had wrote to an eminent professor of mathematics to take me into his house, and give me board and lodging, with all proper instructions to qualify me for teaching a mathematical school he (Mr. Poyntz) had in view for me, and would get me settled in it. This I should have liked very well, especially as I began to be tired of drawing pictures, in which, I confess, I never

stroved to excel, because my mind was still pursuing things more agreeable. He soon after told me he had just received an answer from the mathematical master, desiring I might be sent immediately to him. On hearing this, I told Mr. Poyntz, that I did not know how to maintain my wife during the time I must be under the master's tuition. What, says he, are you a married man? I told him I had been so ever since May in the year 1739. He said he was sorry for it, because it quite defeated his scheme; as the master of the school he had in view for me must be a batchelor.

He then asked me what business I intended to follow? I answered, that I knew of none besides that of drawing pictures. On this he desired me to draw the pictures of his lady and children, that he might shew them in order to recommend me to others; and told me, that, when I was out of business, I should come to him, and he would

would find me as much as he could: and I soon found as much as I could execute: but he died in a few years after, to my inexpressible grief.

Soon afterward, it appeared to me, that, although the Moon goes round the Earth, and that the Sun is far on the outside of the Moon's orbit, yet the Moon's motion must be in a line that is always concave toward the Sun: and upon making a delineation representing her absolute path in the Heavens, I found it to be really so. I then made a simple machine for delineating both her path and the Earth's on a long paper laid on the floor. I carried the machine and delineation to the late Martin Folkes, Esquire, President of the Royal Society, on a Thursday afternoon. He expressed great satisfaction at seeing it, as it was a new discovery; and took me that evening with him to the Royal Society, where I shewed the delineation, and the method of doing it.

When

When the business of the Society was over, one of the members desired me to dine with him next Saturday at Hackney; telling me that his name was Ellicott, and that he was a watch-maker.

I accordingly went to Hackney, and was kindly received by Mr. John Ellicott, who then shewed me the very same kind of delineation, and part of the machine by which he had done it; telling me that he had thought of it twenty years before. I could easily see, by the colour of the paper, and of the ink lines upon it, that it must have been done many years before I saw it. He then told me, what was very certain, that he had neither stolen the thought from me, nor had I from him. And from that time till his death, Mr. Ellicott was one of my best friends. The figure of this machine and delineation is in the 7th Plate of my book of Astronomy.

Soon after the stile was changed, I had my Rotula new engraved; but have

have neglected it too much by not fitting it up and advertising it. After this, I drew out a scheme, and had it engraved, for shewing all the problems of the Rotula except the Eclipses: and, in place of that, it shews the times of rising and setting of the Sun, Moon, and Stars; and the positions of the Stars for any time of the night.

In the year 1747, I published a Dissertation on the Phenomena of the Harvest Moon, with the description of a new Orrery, in which there are only four wheels. But having never had grammatical education, nor time to study the rules of just composition, I acknowledge that I was afraid to put it to the press; and, for the same cause, I ought to have the same fears still. But having the pleasure to find that this my first work was not ill received, I was emboldened to go on, in publishing my Astronomy, Mechanical Lectures, Tables and Tracts relative to several Arts and Sciences, *The Young Gentleman*

Gentleman and Lady's Astronomy, a small treatise on Electricity, and the following sheets.

In the year 1748, I ventured to read Lectures on the Eclipse of the Sun that fell on the 14th of July in that year. Afterwards I began to read Astronomical Lectures on an Orrery which I made, and of which the figures of all the wheel-work are contained in the 6th and 7th Plates of this book. I next began to make an apparatus for Lectures on Mechanics, and gradually increased the apparatus for other parts of Experimental Philosophy, buying from others what I could not make for myself, till I brought it to its present state.—I then entirely left off drawing pictures, and employed myself in the much pleasanter business of reading Lectures on Mechanics, Hydrostatics, Hydraulics, Pneumatics, Electricity, and Astronomy: in all which, my encouragement has been greater than I could have expected.

The best machine I ever contrived is the *Eclipsareon*, of which there is a figure in the 13th Plate of my Astronomy. It shews the time, quantity, duration, and progress of Solar Eclipses, at all parts of the earth. My next best contrivance is the Universal Dialing Cylinder, of which there is a figure in the 8th Plate of the Supplement to my Mechanical Lectures.

It is now thirty years since I came to London ; and during all that time, I have met with the highest instances of friendship from all ranks of people both in town and country, which I do here acknowledge with the utmost respect and gratitude ; and particularly the goodness of our present gracious Sovereign, who, out of his privy purse, allows me fifty pounds a-year, which is regularly paid without any deduction.

E R R A T A;

Pag. 73. l. 4. for fifth read sixth. Pag. 187. l. 8. in the Table, in the second column, for 149.9 read 194.9. Pag. 242. l. 4. from the bottom, for plane read place. Pag. 261. l. 14. for roused read raised.

Note, in Page 244. and afterward throughout, for Napier read Nepair; as that was the way in which the Baron of Merchiston himself spelt his Name.

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